Convex Polytiles Enumerations (part 1b) Zonogons, Progons, and Exotics

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Abstract. *A **polytile** or p-tile as an equilateral polygon with turn angles as multiples of 360°/p, for even p=4, 6, 8…. This paper continues from paper 1a introducing polytiles. Section 2 explores the enumeration for strictly convex p-tiles, and subcounts by sides. Second 4 describes software used to explore polytiles. Section 5 offers tables for all solutions up to 18-tiles. Section 6 enumerations polytile counts by sides and symmetry to 32-tiles. Section 3 shows most of **zonogons (**even-sided with parallel opposite edges). Equilateral zonogons allow rhombic dissections, and the **Generalized Dual Method (GDM)** allows translated dual edge lines intersections to define a rhombic dissection. We also define a new class of **progons,** generalizing zonogons, as Minkowski sums of regular polygons instead of just digons (vectors). Regular polygons have regular k-rays duals (k divisor of p) making k-belts of rhombi. Zonogon and progons can be seen as projective envelopes of prism products of regular polygons. **Exotic** polytiles appear at 30-tiles which can't all be decomposed into independent k-belts, requiring adjacent triangles and pentagons to dissect.*

1 Introduction

Polytile were first defined in part 1a in 2021 [5], showing a wide range of applications of polygons that can be defined with integer turn angles. This paper focuses on convex polytiles, while many of the ideas can extend to polytiles with colinear edges and concavities.

You can explore immediately with the online application Polytile Explorer, with a short description in section 5. Hyperlinks within this document will open Explorer with specific polytiles. https://www.bendwavy.org/polytile/drawtile.html

Polytile definition

A polytile or p-tile is an equilateral polygon with special angles. Vertex turn angles are defined as integers and represent 360°a_i/p.[5]

Polytile notation has the form: ***p*:*a*₁.*a*₂.*a*₃…*a*ₖ^*n***, repeating angles **$a_1.a_2...a_k$** *n* times. *p*=4,6,8,10,… For compactness, the delimiter can be assumed if they are all single digits.

The a_i angles for a convex polytile are between 1 and *p*/2-1. An angle of *p*/2 could be included as a digon. A straight vertex is defined with an angle of 0. A concave polytile can have negative indices as reverse turn angles.

For simple (single turn) polytiles, the **p:** is implicitly determined by the sum of the a_i indices, $p=n(a_1+a_2+…+a_k)$. For example, **1.2^2** = **12^2** is a rhombic hexatiles (*p*=6), explicitly **6:1.2^2** or **6:1212**.

The starting vertex angle doesn't matter for a cyclic sequence. For example: **123^2** = **231^2** = **312^2**.

Parentheses can nested and grouping sequences of repeated angles, like **12:2.(1^4)^2** is the same as **2.1.1.1.1^2** which is the same as **2.1.1.1.1.2.1.1.1.1**.

Geometric interpretation

The *p*-tile construction has a geometric interpretation, using regular *p*-gons attached edge-to-edge, with the polytile vertices defined by the center of the regular *p*-gons, and the edges of the polytiles defined by a center to edge to center distance between adjacent *p*-gons. Since this length is fixed for a given *p*-gon, the polytiles are equilateral.

Figure 1 shows four examples convex dodecatiles defined by their cyclic notation and partition notation. In the center of each polytile is its Conway polyhedron symmetry, described below.

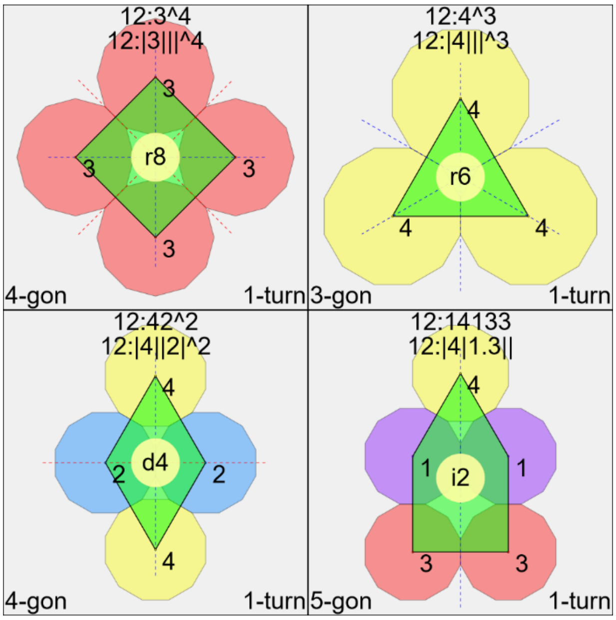


Figure 1 Example dodecatiles defined by edge-to-edge dodecagons in a cycle.

Conway polygon symmetry

Conway polygon symmetry expresses the cyclic or dihedral symmetry of a polygon with a letter and a group order. The letter gives further information for where mirror planes exist relative to the vertices or edges. A regular n-gon is given symbol $\mathbf{r_{2n}}$, **r** standing for regular, and having D_n dihedral symmetry, order 2n. A polygon with no reflectional symmetry is given as $\mathbf{g_n}$, **g** standing for gyration, having cyclic symmetry C_n. A polygon with no symmetry is $\mathbf{a_1}$, **a** standing for asymmetric. Polygons with reflectional symmetry have 3 possible letters, **p**, **i**, and **d**, **p** standing for perpendicular (reflections passing through mid-edges), **i** for intermediate (mirrors passing through both vertices and edges), and **d** for diagonal (mirrors passing only through vertices). Reflective symmetry polygons always have dihedral symmetry, and, like regular forms, are only of even order.

Partition notation simplifies *polytile notation* for cases with dihedral (reflective) symmetry to avoid repeating a reversed direction sequence.

There are 3 forms, depending on whether vertices or edges cross the reflection lines:

- $\mathbf{p{:}|b|a_1.a_2.a_3\ldots a_k|c|{}^\wedge n = p{:}b.(a_1.a_2.a_3\ldots a_k).c.(a_k.a_{k-1}.a_{k-2}\ldots a_1){}^\wedge n}$ polygons with reflections crossing vertices with turn angles **b** and **c**.
- $\mathbf{p{:}|b|a_1.a_2.a_3\ldots a_k||{}^\wedge n = p{:}b.(a_1.a_2.a_3)\ldots(a_k.a_k.a_{k-1}.a_{k-2}\ldots a_1){}^\wedge n}$ … polygons with reflections crossing vertices with angle ***b***, and edge $\mathbf{a_k.a_k}$.
- $\mathbf{p{:}||a_1.a_2.a_3\ldots a_k||{}^\wedge n = p{:}(a_1.a_2.a_3\ldots a_k).(a_k.a_{k-1}.a_{k-2}\ldots a_1){}^\wedge n}$ … *polygons with reflections crossing edges* $\mathbf{a_1.a_1}$ *and* $\mathbf{a_k.a_k}$.

Properties of polytiles

$\mathbf{p{:}a_1.a_2.a_3\ldots a_k{}^\wedge n}$ and $\mathbf{p{:}a_k.a_{k-1}.a_{k-2}\ldots a_1{}^\wedge n}$ are chiral pairs if distinct. Otherwise, it has dihedral (reflective) symmetry.

With the highest ***n,*** a polytile has *n*-fold cyclic (rotational) symmetry.

If the highest ***n*** *is* **1**, it is asymmetric. A general sequence $\mathbf{p{:}a_1.a_2.a_3\ldots a_k}$ in general defines an open path (called a **polychain**), rather than a closed polygon, but some sequences will close.

If n is even, the polytile is also an **equilateral zonogon**, a 2-dimensional case of a **zonotope**, containing central symmetry, having an even number of sides, and can be seen.[3]

Properties of a simple strictly convex p-tile:

- A convex p-tile has most p vertices or corners, p edges or sides. A regular p-gon is the highest case: p:1^p, constructed with p (360°/p) turns.
- A convex polytile, $p{:}a_1.a_2.a_3\ldots a_k$^$n$, *with* $p=n(a_1+a_2+\ldots+a_k)$, always exists if $n>2$. If n is a multiple of 2, the polytile is a zonogon.

Haresh Lalvani

Since (Polytiles part 1) [5] was published in 2021, I've been made aware of Architect Haresh Lalvani[4] who in 1998 designed a very similar geometric system as polytiles, specified as integers corresponding to internal angles, as integers between 1 and p-1, with 1 as the sharpest angle and p/2-1 as the shallowest angle, and angles greater than p/2 make concave vertices. With internal angles summing to p, this allows a vertex to be filled. He also draws regular polygons at the vertices, but his tiles are named by an ID number rather than angle sequences.

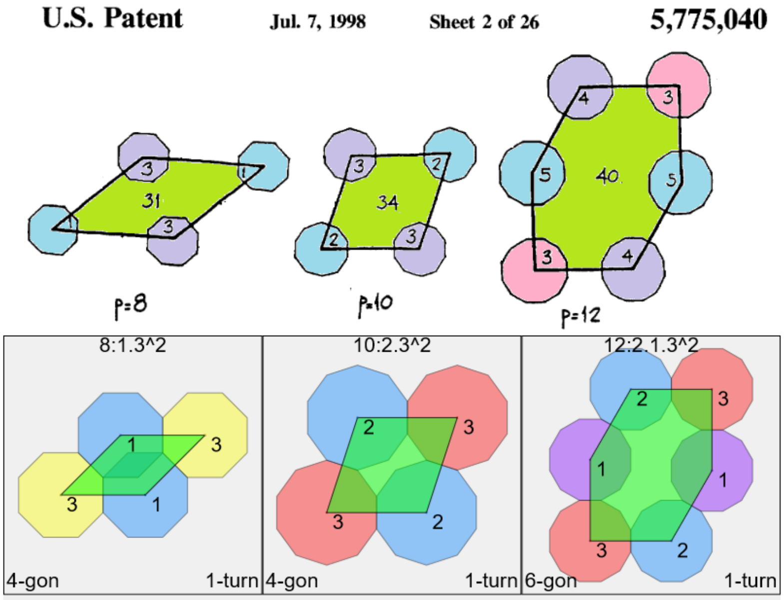


Figure 2: Haresh Lalvani drawings above, polytiles below.

Figure 2 shows 3 tiles in Lalvani's drawings and compares to polytiles, 31^2, 32^2, and 321^2.

2 Solution search algorithm

A strictly convex p-tile has the form $\boldsymbol{p{:}a_1.a_2.a_3\ldots a_k}$, repeating angles $\boldsymbol{a_1\ldots a_k}$, with $\boldsymbol{p=a_1+a_2+\ldots+a_k}$, with turn angles a_i between 1 and $p/2-1$. The number of vertices, k, is at most p in the final regular case.

A simple brute search can iterate all possible values for a_i as nested loops, with the fewer than p vertices by defining a last angle can be computed by the desired p, and other angles: $a_k = p - (a_1+a_2+\ldots+a_{k-1})$ if this value is less than $p/2$.

Once a polytile $\boldsymbol{p{:}a_1.a_2.a_3\ldots a_k}$ is identified in the search, it must be compared to previous solutions. This could be an exact match of the sequence, or as a shift of k possible starting indices, or as a reversed list and shifted starts. If a p-tile is unequal to its reverse, it has chiral symmetry, and exists in chiral pairs. This enumeration will consider these identical solutions.

A p-tile reduced to a form $p{:}a_1.a_2.a_3\ldots a_k$^$n$ has n-fold rotational symmetry, with no smaller repeats in $a_1.a_2.a_3\ldots a_k$. If $k \geq 2$, the context p-tile is guaranteed to close. For k=1, it must be explicitly constructed as a vector sum (algorithm 1) to check if it closes.

Solutions

A summary table for p-tiles by sides is give here, up to *p*=40.

Trivially only triangle solutions are equilateral triangles for *p*=6, 12, 18, 24… The quadrilateral solutions are squares and rhombuses. Square are with *p*=4, 8, 12… One extra rhombus is added every 4*p*.

Table 1: Solution counts by p-tile rows and counts by sides in columns.

p	Total	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
4	1		1																																				
6	3	1	1		1																																		
8	4		2		1		1																																
10	7		2	1	2		1		1																														
12	16	1	3	1	4	1	3	1	1		1																												
14	17		3		4	1	4		3		1		1																										
16	28		4		5		8		5		4		1		1																								
18	70	1	4	1	9	4	12	6	12	4	9	1	4	1	1		1																						
20	85		5	1	8	1	16	2	17	2	16	1	8	1	5		1		1																				
22	125		5		10		20		26	1	26		20		10		5		1		1																		
24	392	1	6	2	15	8	33	20	50	27	66	27	50	20	33	8	15	2	6	1	1		1																
26	379		6		14		35		57		76	1	76		57		35		14		6		1		1														
28	704		7		16	1	47	1	79	3	126	4	134	4	126	3	79	1	47	1	16		7		1		1												
30	3359	1	7	3	24	17	71	60	173	145	329	249	442	315	442	249	329	145	173	60	71	17	24	3	7	1	1		1										
32	2248		8		21		72		147		280		375		440		375		280		147		72		21		8		1		1								
34	4111		8		24		84		196		392		600		750	1	750		600		392		196		84		24		8		1		1						
36	18510	1	9	3	32	21	116	90	322	260	775	599	1384	1073	2024	1392	2306	1392	2024	1073	1384	599	775	260	322	90	116	21	32	3	9	1	1		1				
38	14309		9		9		30		120		324		756		1368		2052		2494	1	2494		2052		1368		756		324		120		30	0	9	0	1		
40	30820	0	10	1	33	2	145	14	409	49	1036	147	2001	315	3300	521	4340	667	4838	667	4340	521	3300	315	2001	147	1036	49	409	14	145	2	33	1	10	0	1	0	1

Extrapolation of counts

The number of convex p-tiles is approximately linear on a log graph, with 6p-tiles having more than others. The 6*p*-tile sequence nicely fits with 0.4*6^(*p*/6), seen in red line. This extrapolates over 100,000 solutions for *p*=42. A more efficient search algorithm and a little more time would allow that to be determined exactly.

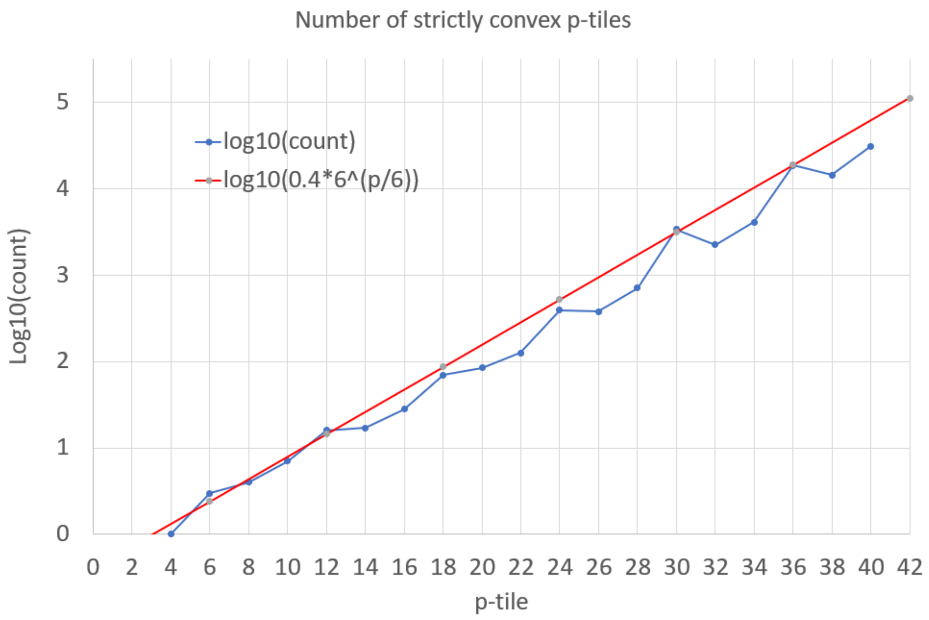


Figure 3(a) Log10 graph of counts vs p-tiles to 40

Figure 3(a) shows an exponential rise in counts with p, with peaks at multiples of 3. Figure 3(b) shows counts for only zonogons which is smoother, but still not quite linear on a log graph

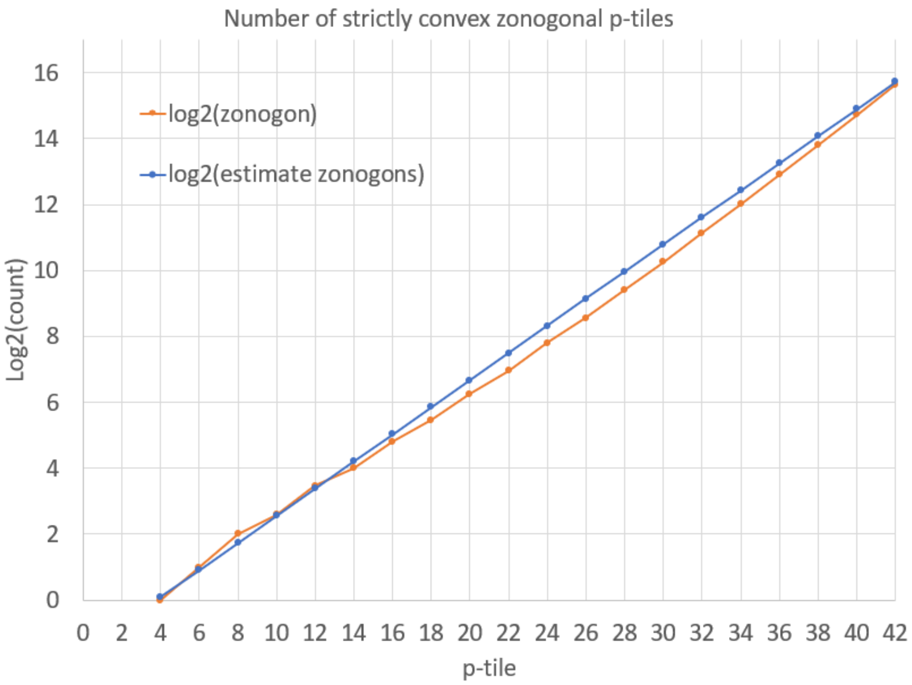


Figure 3(b) Log2 graph of counts of zonogons p-tiles to 42.

3 Structure of Polytile

Symmetry of a polytile is an important structure of a polytile. Cyclic symmetry, order *n*, is expressed in polytile notation by the exponent: $p{:}a_1\ldots a_k$^n, but this is not all symmetry that can be extracted.

Zonogons

Polytiles with an even cyclic order are zonogons, with central (2-fold) symmetry. A convex polytile p:$a_1\ldots a_k$^2n is a 2nk-zonogon. Pairs of opposite edges are parallel and associated together as "belts". Rhombic dissections of zonogons propagate sequences of rhombi with parallel opposite sides across the interior. A 2n-zonogon has n vector orientations, and their Minkowski sum expresses the perimeter of the polytile, while partial sums produce a given dissection of the interior into rhombi.[3]

Figure 4 shows rhombic dissections for the 11 zonogonal dodecatiles, labeled above as n{2} for the Minkowski sum of n vectors, as digons {2}. Edges are given 6 colors based on their orientations.

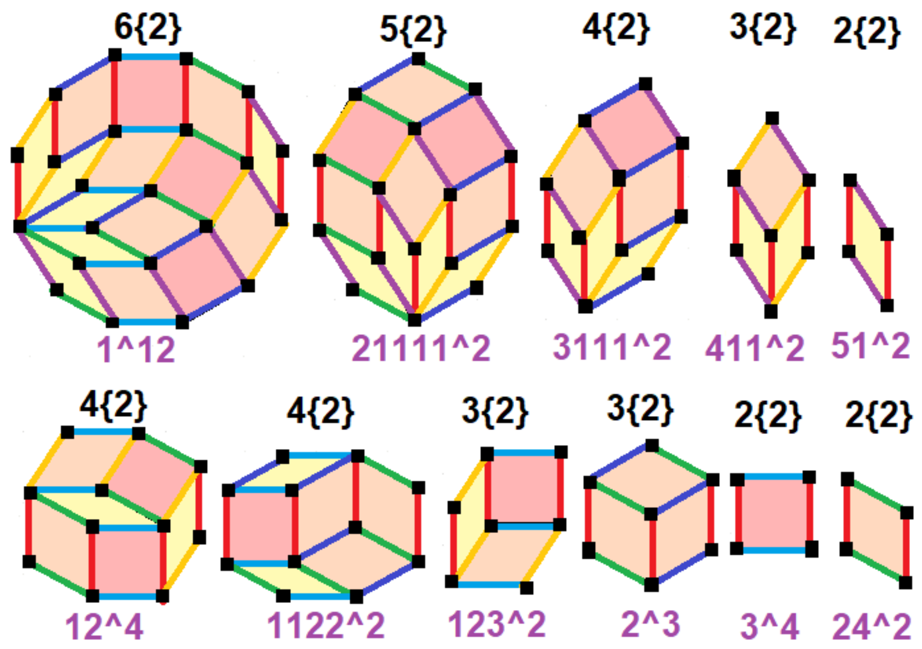


Figure 4 Rhombic dissection of 11 zonogonal dodecatiles

Dual vectors and Generalized Dual Method

Dissections of a convex polytile can be expressed in a dual network of intersecting lines. This is done via the **Generalized Dual Method**.[6] A dual polygon is first constructed by a set of rays or vectors from the center and normal to a given edge and extending to infinity. In a zonogon, pairs of opposite rays are combined and replaced by a line. These lines can then be translated away from a single center. Each pairwise intersection of those lines defines a rhombus with angles equal to the angle between the two intersecting lines. These rhombi are attached together edge-to-edge as defined by the network of intersecting lines.

Figure 5 Left shows a regular decagon first with 10 dual rays grouped into dual lines of a zonogon. The middle shows these 5 lines translated in a symmetric way into a dual network, producing 10 intersections, with wide and narrow rhombi drawn on top to show their related rhombi. Right shows the corresponding rhombic dissection of the decagon when the rhombi are pulled together edge-to-edge. Edges of the rhombi are again colored to match their dual lines.

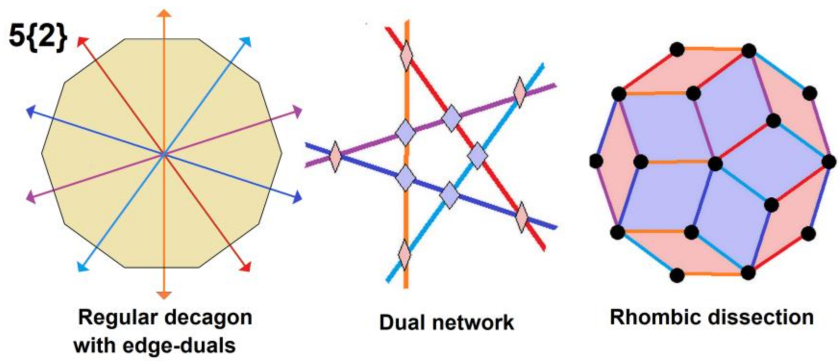


Figure 5 Example symmetric rhombic dissection of a regular decagon

Figure 6 shows a regular dodecagon {12}, decomposed symmetrically with a dual network.

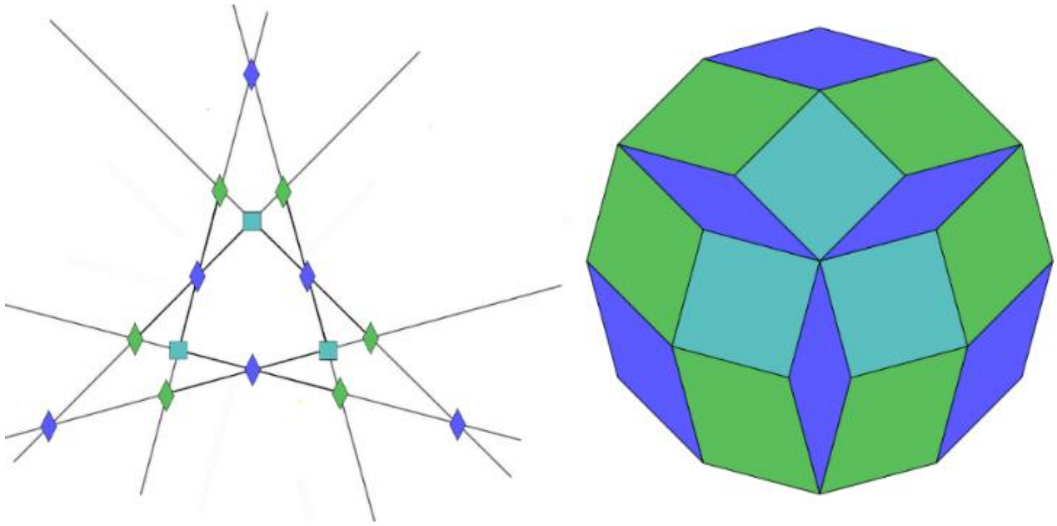


Figure 6 Symmetric rhombic dissection of a regular dodecagon

Progons as generalized zonogons

We can use the **Generalized Dual Method** in polytiles that are not zonogons. At minimum we need to add prime-sided regular polygons to our dissections. However, we can also keep all regular polygons as a simplest dissection and subdivide the compose regulars second. This generalizes “belts” in zonogons into k-belts with k-ray duals. A dissection has a central regular polygon radiating outwards into belts of rhombi out to a perimeter edge.

Figure 7 shows dual k-rays for regular polygons 2…9. The 2-gon, or digon is degenerate, but can be seen as 2 coinciding edges and zero area. The vectors can be called k-rays for a regular k-gon.

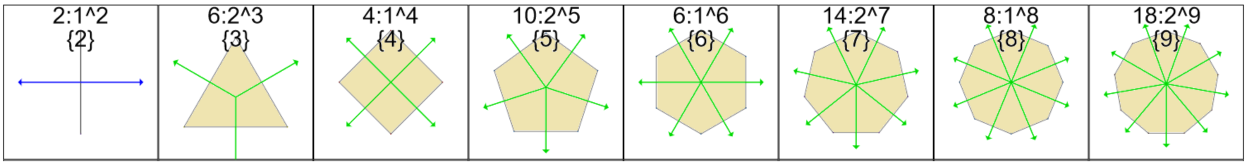


Figure 7 Dual vectors of regular polygons of 2 to 9 sides

Definition: A **progon** is the Minkowski sum of regular polygons. Zonogons are a subclass of progons summing only digons, {2} as vectors. There can be more than one decomposition for a given progon. I named this generalization a progon as a “product polygon”, produced by a set of smaller polygons. In 2D this is called a Minkowski sum (perimeter edges sum), but each can also be seen as the projective envelop of a higher dimensional prism, also called a proprism by John Conway.[2] A specific dissection can be seen as a subset of faces of that projected proprism.

Figure 8 shows a regular hexagon and decagon can be decomposed as 3{2} and 5{2} as zonogons, but also 2{3} and 2{5} as more general progons. The dual vectors are 2 3-rays, 3 2-rays, 2 5-rays, and 5 2-rays respectively.

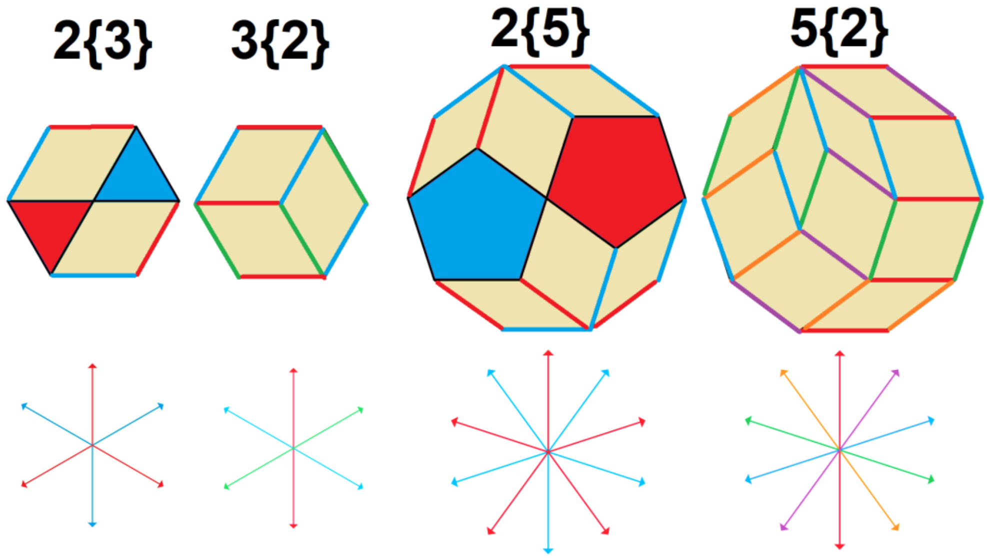


Figure 8 Decompositions of hexagon and decagons with k-rays

Minkowski sums of regular polygons

It is a surprising idea, that convex polygons can be added together. In fact, any two convex polygons can be added while we're just limiting to regular polygons for our application of dissecting polytiles.

Figure 9 shows 2{3} = {3} + {3} = {6}. The decomposed 2 3-rays can be translated to define a dissection of the hexagon as 2 triangles and 2 rhombi. However due to the anti-parallel vectors, it is possible to make them coincide, which creates an unexpected dissection of 2 trapezoids, sharing a double length middle edge. This is allowed, but also easily removed by an infinitesimal translation, restoring the expected two pairwise intersections again.

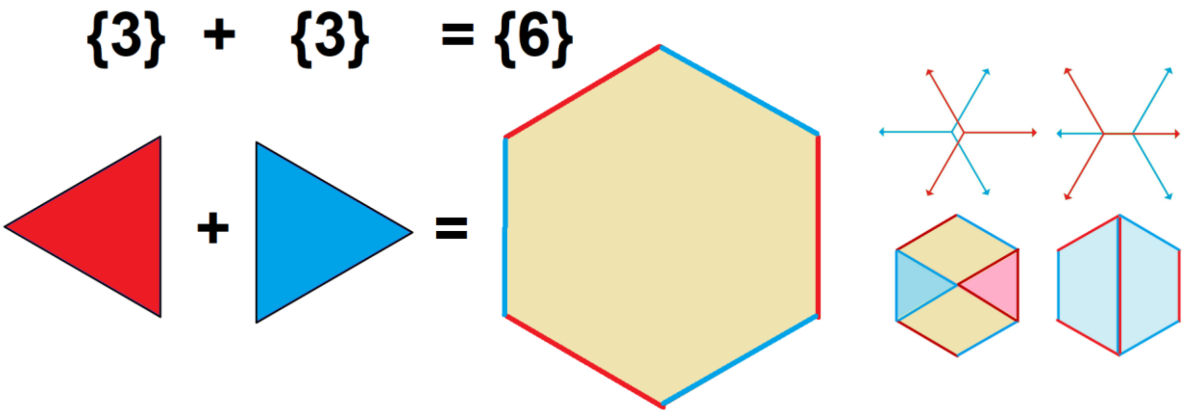


Figure 9 A hexagon is a Minkowski sum of two triangles

Projections of higher dimensional prisms

Zonogons have another interpretation as projections of n-cubes into the plane, with each rhombus seen as a square in higher dimensions projected at a skew angle. An n-cube is more generally a product of orthogonal digons {}^n in n-dimensional space.

Progons work similarly, with each digon {2} existing in 1D, and each polygon {p} in 2D.

Figure 10 shows an example of a triangle and hexagon summed as a convex 9-gon, also polytile 12:112^3. The Minkowski sum expresses the new outer perimeter, while the full dissection can be seen as a projection of a {6}x{3} duoprism in 4-dimensions. For clarity the dissection is drawn twice, second-most-right with 3 (yellow) hexagons with triangular translation, and right showing 6 triangles by hexagonal translation. The union of these set of translated hexagons and triangles defines the polygon interior.

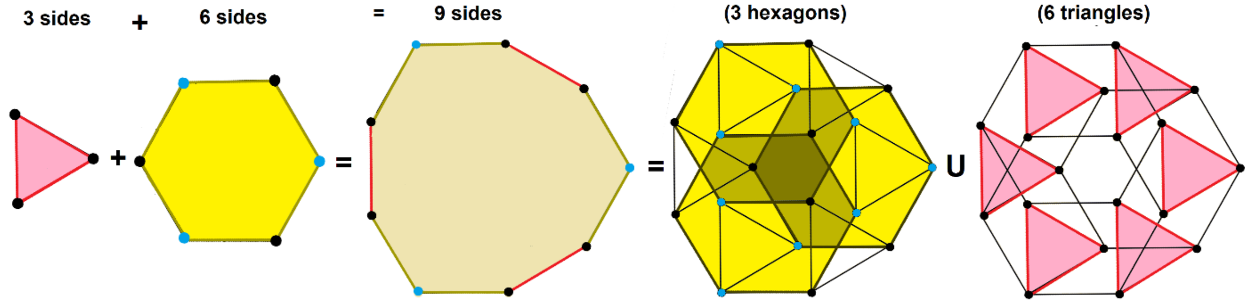


Figure 10 Minkowski sum of a triangle and hexagon

Exotic decompositions

Not all polytiles can be dissected into independent regular polygons in a Minkowski sum. The exceptional cases are easily identified as dissections with 2 odd-sided polygons meeting edge to edge. I call these **exotic**, misusing the ex-ot sound to imply them as “extra odd”.

Pairs of triangles edge to edge making a rhombic perimeter is the simplest case. This result cannot be expressed as a Minkowski sum of the two triangles but something different.

Figure 11 shows a 30-tile example, 30:1.6.6.6.1.10 which has dual vectors which cannot be dissected into two independent k-rays, but we easily see on this simple case, that it can be dissected into a triangle and pentagon. And the dual vectors show sort of composite of a 3-ray and 5-ray, with a common ray reduced into an internal edge that doesn’t appear in the outer rays.

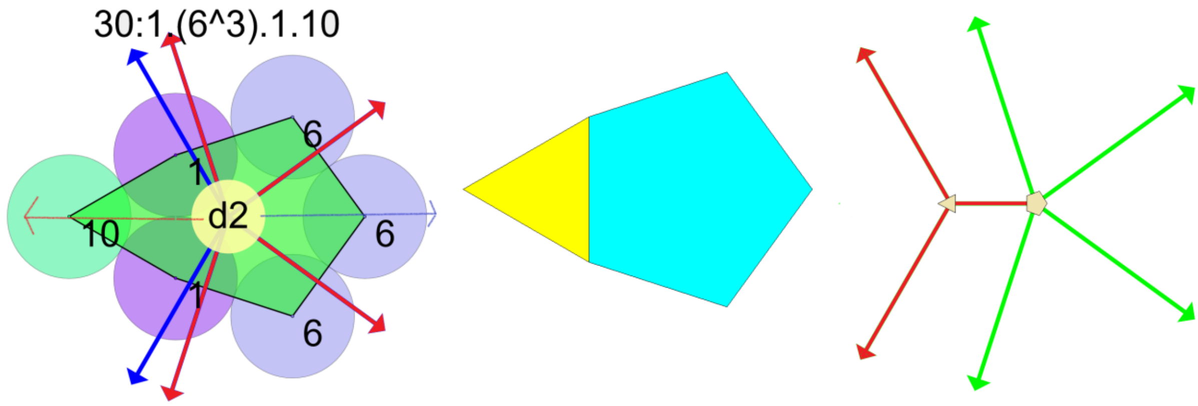


Figure 11 An exotic tile without independent decompositions of regular polygons

Further classification of these exotic polytiles could be done, but they are more difficult to deduce in more complicated cases. And as dual networks they can interact with progon k-rays perfectly well, generating rhombic tiles at pairwise intersections.

Variations of decomposition

The more divisors in the cyclic symmetry of a polytile, the more possibilities there are Minkowski decompositions. The regular dodecagon {12} can be dissected as 6{2} (as a zonogon in figure 6), as well as 3{4}, 3{2}+{6}, 2{6}, 2{3}+[6], and 4{3}.

Figure 12 shows 6 decompositions from a regular dodecagon, {12}, with 6 example dissections. The right most dissection includes an exotic "benzene ring" network which enables the 6 outer blue triangles. The decomposed polygons in row 3 are colored to match the k-rays. For fun, row 4 show all of the dissections can be done with Pattern Block. [1]

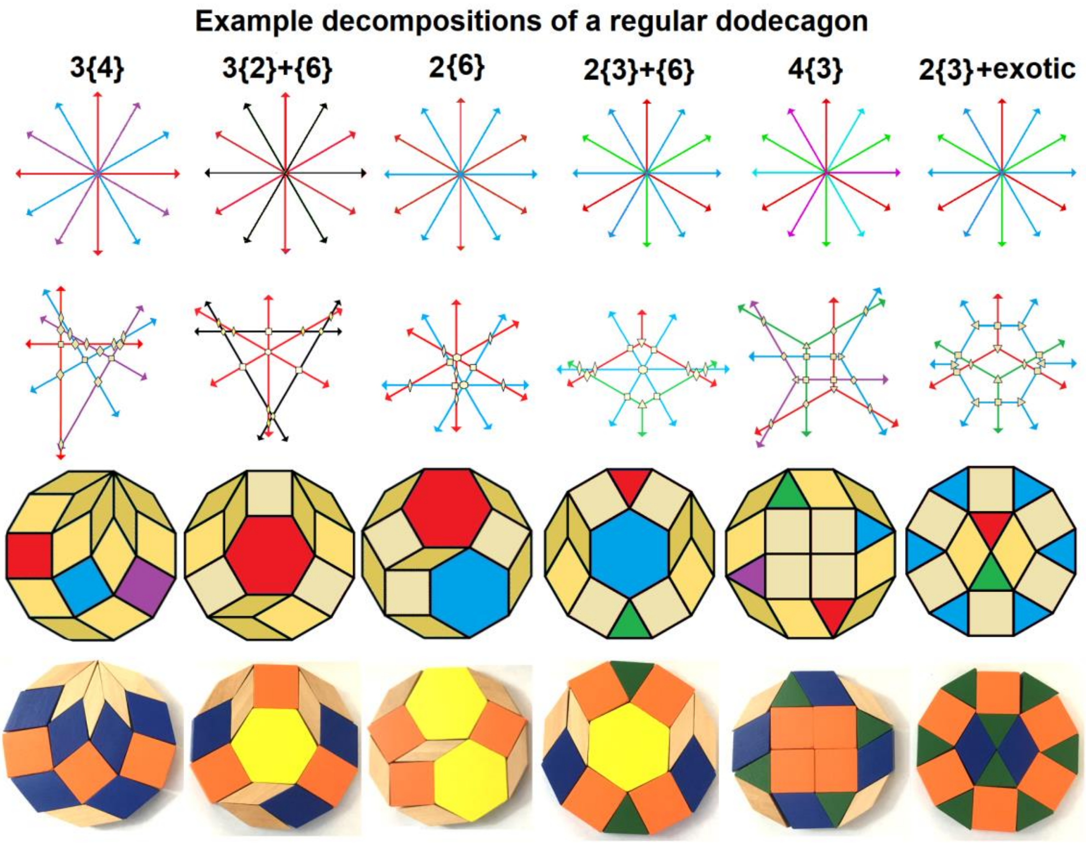


Figure 12 Decompositions of a regular dodecagon

Decomposing algorithm

So far we've looked at regular polytiles which are easy to decompose. For other polytiles, we have to make more decisions how to decompose.

The tables in the section 5 show Minkowski sum decompositions by searching the dual vectors for the even-sided k-rays first, largest down to smallest 2, with k a divisor of p, marking vectors after extraction so they cannot be reused. Then I search odd-sided k-rays, largest down to smallest, with k divisors of p. The process stops until all dual vectors are marked.

If the process stops with any still unmarked vectors, I reset and try again, but search odds first, and evens second. If that fails, I call the polytile exotic.

All polytiles below 30-tiles can be classified as progons, with at least one decomposition of the dual vectors into *k*-belts. For 30-tiles, 481 are identified as exotic (not listed in this paper). A more complete search might find some that I name exotic could have been decomposed for instance with 3-rays done before 5-rays.

Two example decompositions of nonregular polytiles

Figure 13 shows a dodecatile heptagon, 12:1312131. It dissects into one 3-ray (triangle) and two 2-rays, Minkowski sum 2{2}+{3}. The "full dissection" can be seen as a projection of a duoprism, {4}x{3}, with squares are projected as rhombi. The full dissection shows 4 translated triangles, and 3 skew-squares, interconnected with rhombi. On the right are 4 unique dissections.

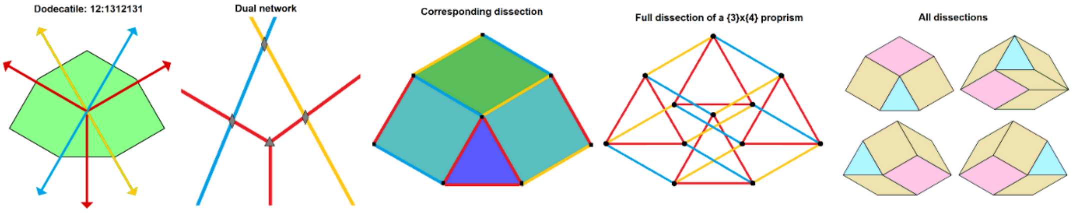


Figure 13 Decomposition of a dodecatile into a 3-ray and 2 2-rays.

A more complex example is shown in figure 14, polytile 18:13122112311 with no global symmetry. But a k-ray decomposition search extracts one 6-ray, one 2-ray, and a 3-ray remaining. So this funny polytile decomposition is {2}+{3}+{6}.

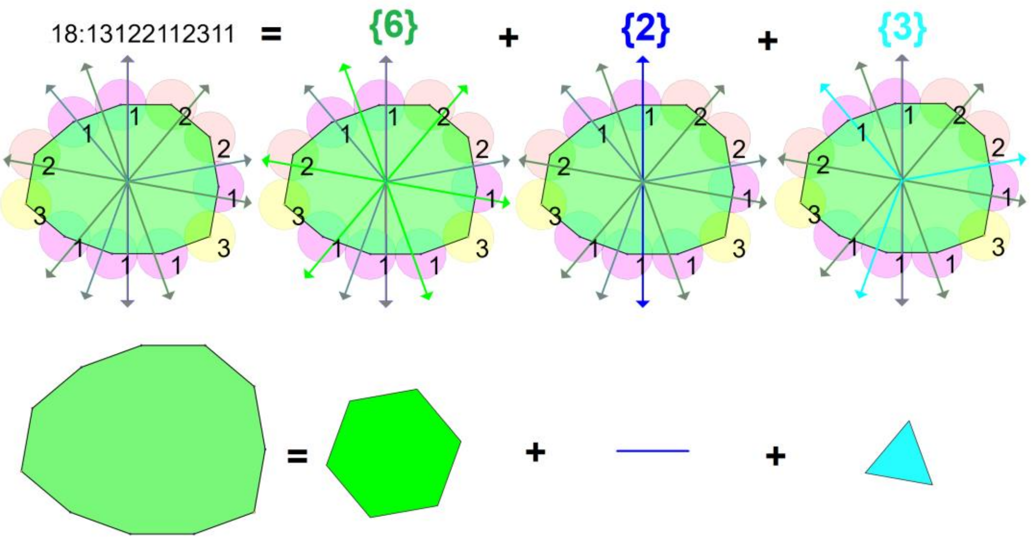


Figure 14 Decomposition of an asymmetric 18-tile

A hierarchy of reduced polytiles

A powerful fact about zonogons and progons is each k-ray is independent and can be removed from the dual network. That removal effectively reduces the polytile by k edges, and this reduction can be done in any order, so you could make a graph of reduced polytiles from one larger tile decomposition. A polytile with 3 decomposition elements will have 7 Minkowski forms (2^3-1) whether each element is included or excluded.

For figure 15 shows example 12:21111^2 can be decomposed as 2{}+{6}, and {6} can be decomposed as 2{3}, we can express it as 2{2}+2{3}. Keeping rhombus 2{2} as a polygon, we can selectively remove elements to the Minkowski sum. I use a specific dissection to help me show the reduced sums, selectively removing a primary polygon, and all of its belt rhombi (coloring them gray)

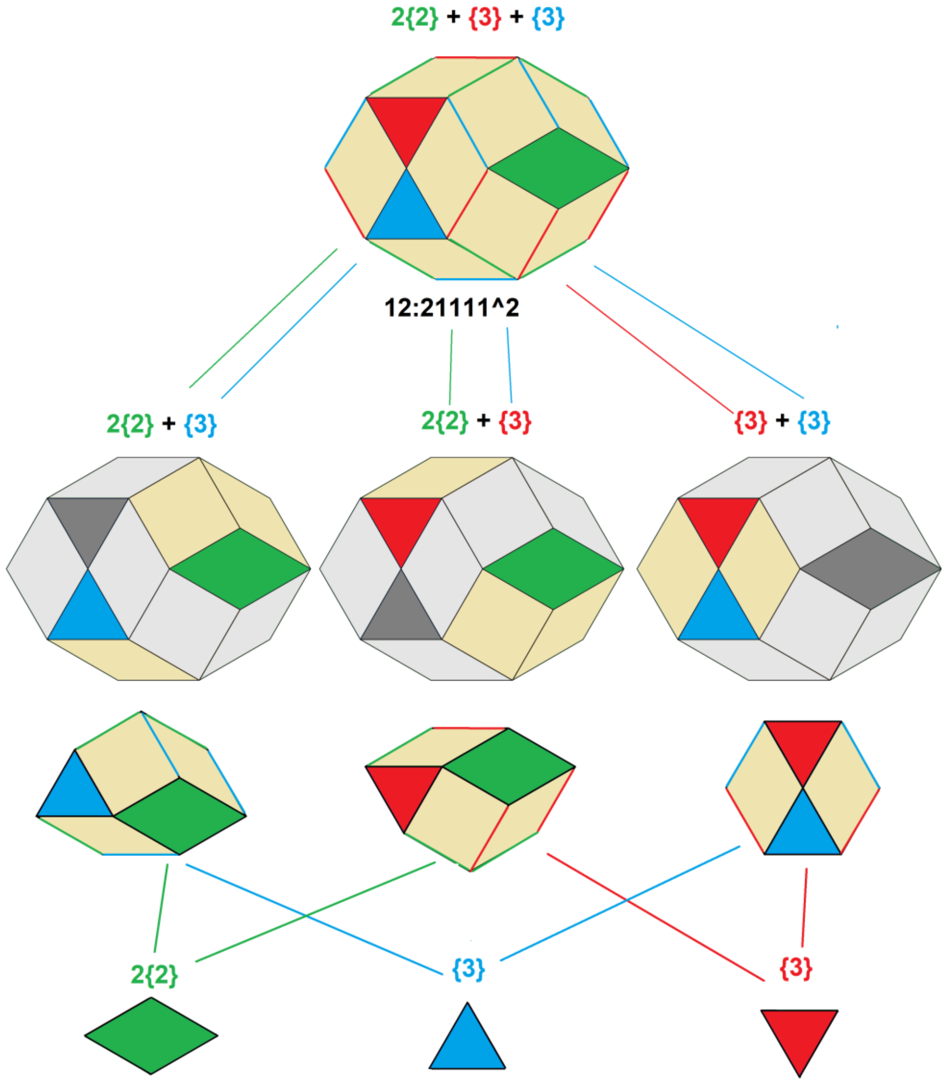


Figure 15 A dodecatile as a Minkowski sum of 3 polygons

4 Software – Polytile Explorer

The polytiles draw in this paper were done through a Javascript App which can be run online in any JS enabled browser: https://www.bendwavy.org/polytile/drawtile.html.

It allows a polytile notation input string to be computed and rendered. There's also a "gallery" mode that shows a set of polytiles into a matrix, like the 16 "convex dodecatiles" show in figure X (left), which is predefined in the "Sets" dropbox. In gallery mode you can click on any image to turn off the gallery and show that polytile alone.

The right side of figure 16 shows a single polytile. You can experiment by typing different notations and see what happens,and press UNDO to go to the last form if you don't like it. You can press "Keep" to add current polytile to the set,and "Remove" to remove it from the set. And "Clear" removes everything from the current set. If you prss "Save", it'll copy the URL along with query parameters for the current set state into the clipboard text.

This long URL includes the convex dodecatile set, and the last one selected.

https://www.bendwavy.org/polytile/drawtile.html?title=Convex_Dodecatiles&code=12:1**12;12:2**6;12:3**4;12:4**3;12:42**2;12:51**2;12:2112**2;12:123**2;12:14133;12:141**2;12:121**3;12:11211**2;12:21**4;12:1311**2;12:31**3;12:1312131&fontsize=80&showdual&polysize=1&showcode&showpolycode&showcount&showsym&showdual&select=16

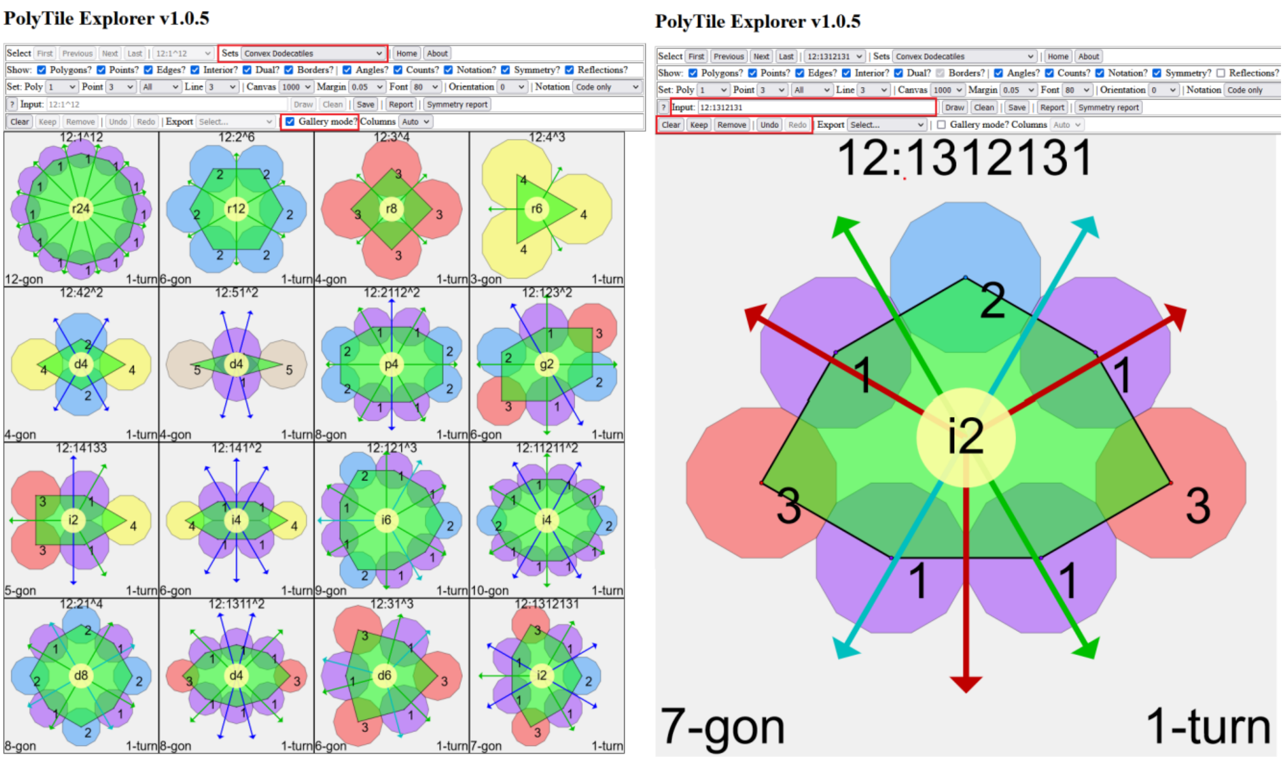


Figure 16 Screen shots from Javascript Application ***Polytile Explorer***

Future Software referenced here:

https://www.bendwavy.org/polytile/

Polytiler

I planned a tiling app where you can input a given tile and click to place. The tiling shown were done with another Java app I wrote, but offline.

Dissection Tool

I also don't have automatic calculations for the Generalized Dual Method for producing specific dissections of the convex polytiles. I made my diagrams by hand simply translating k-ray vector sets in MSPaint, and marking rhombi and regular polygons at the intersections. Drawing the lines by hand really is simple. This feature may be added to Polytile Explorer.

5 Convex p-tile listings

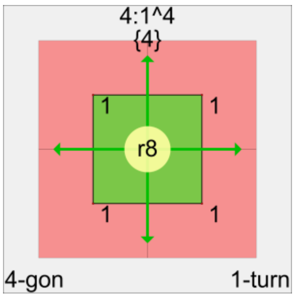
Convex p-tiles for *p*=4, 6, 8, 10, 12, 14, 16, 18, and 20 are given below with their simple codes, $p{:}a_1.a_2\ldots a_m$^n, reflective, code. Vertices ***v***=*mn.*

The **Sym** column gives Conway symmetry notation, with a letter and symmetry order: r=regular, p=perpendicular, i=intermediate, d=diagonal, g=gyro, and a=symmetric.

The ***k*-belt** columns count the number of *k*-belts, where k is a factor of *p*. The **Minkowski sum** give a decomposition of the polytile into smaller regular polygons. **Dim** is the dimension of the related to their corresponding proprism, with each 2-belt existing as one dimension, and higher k-belts existing in 2-dimensions. The tiles are sorted by highest dimension first, and highest vertices second.

Convex Tetratiles

The only convex tetratile is the regular square.

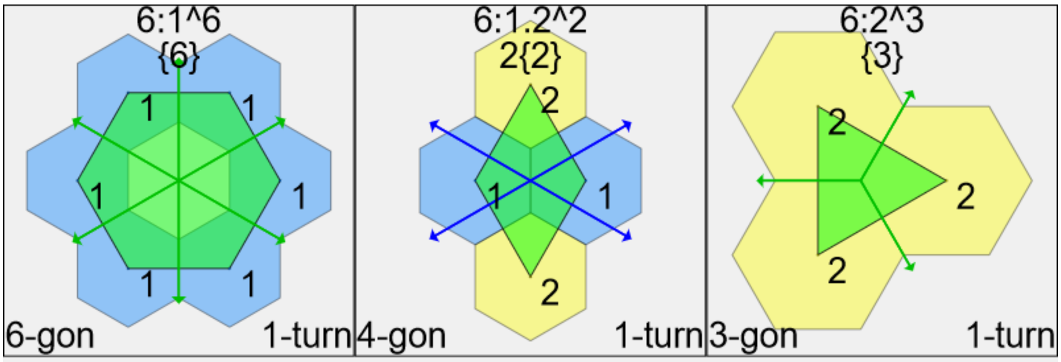


Convex Tetratile

#	Notations		n	m	v	Sym	k-belts	Minkowski	Dim
	Cyclic	Dihedral					4	Sum	
1	4:1^4	4:|1||^4	4	1	4	r8	1	{4}	2

Convex Hexatiles

There are 3 convex hexatiles, including the regular hexagon, triangle, and one rhombus.

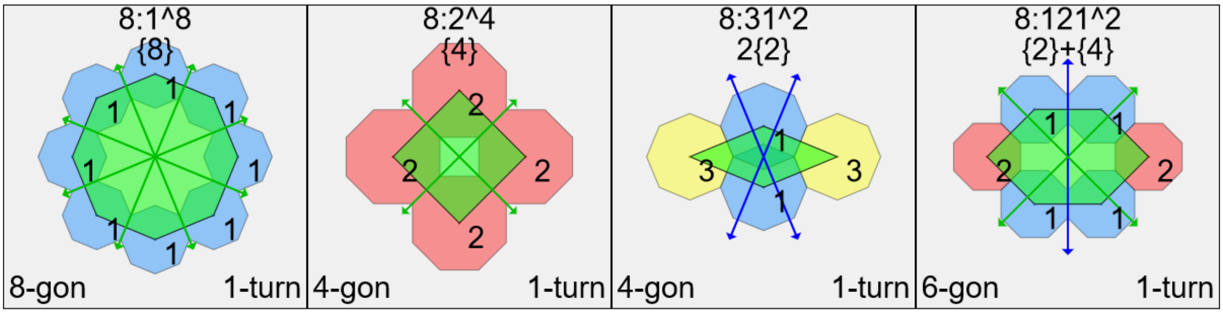


Convex Hexatiles

#	Notations		n	m	v	Sym	k-belts			Minkowski Sum	Dim
	Cyclic	Dihedral					2	3	6		
1	6:1^6	6:|1|||^6	6	1	6	r12	0	0	1	{6}	2
2	6:1.2^2	6:|1||2|^2	2	2	4	d4	2	0	0	2{2}	2
3	6:2^3	6:|2|||^3	3	1	3	r6	0	1	0	{3}	2

Convex Octatiles

There are 4 convex octatiles, including a regular octagon and square. There are two zonogons, a rhombus and hexagon. The hexagon can be decomposed into a Minkowski sum {4}+{2}.

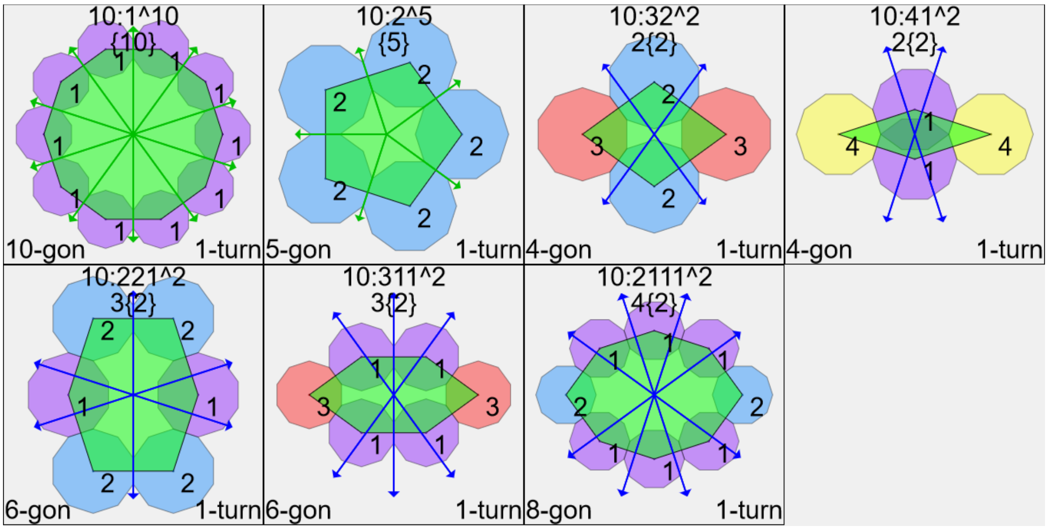


Convex Octatiles

#	Notations		n	m	v	Sym	k-belts			Minkowski Sum	Dim
	Cyclic	Dihedral					2	4	8		
1	8:1^8	8:|1|||^8	8	1	8	r16	0	0	1	{8}	2
2	8:2^4	8:|2|||^4	4	1	4	r8	0	1	0	{4}	2
3	8:31^2	8:|3||1|^2	2	2	4	d4	2	0	0	2{2}	2
4	8:121^2	8:|2|1||^2	2	3	6	i4	1	1	0	{2}+{4}	3

Convex Decatiles

There are 7 convex decatiles, including the regular decagon, pentagon, and 2 rhombi. The rhombi areused Penrose tilings. All 5 nonregulars are zonogons, 4 to 8 sides.

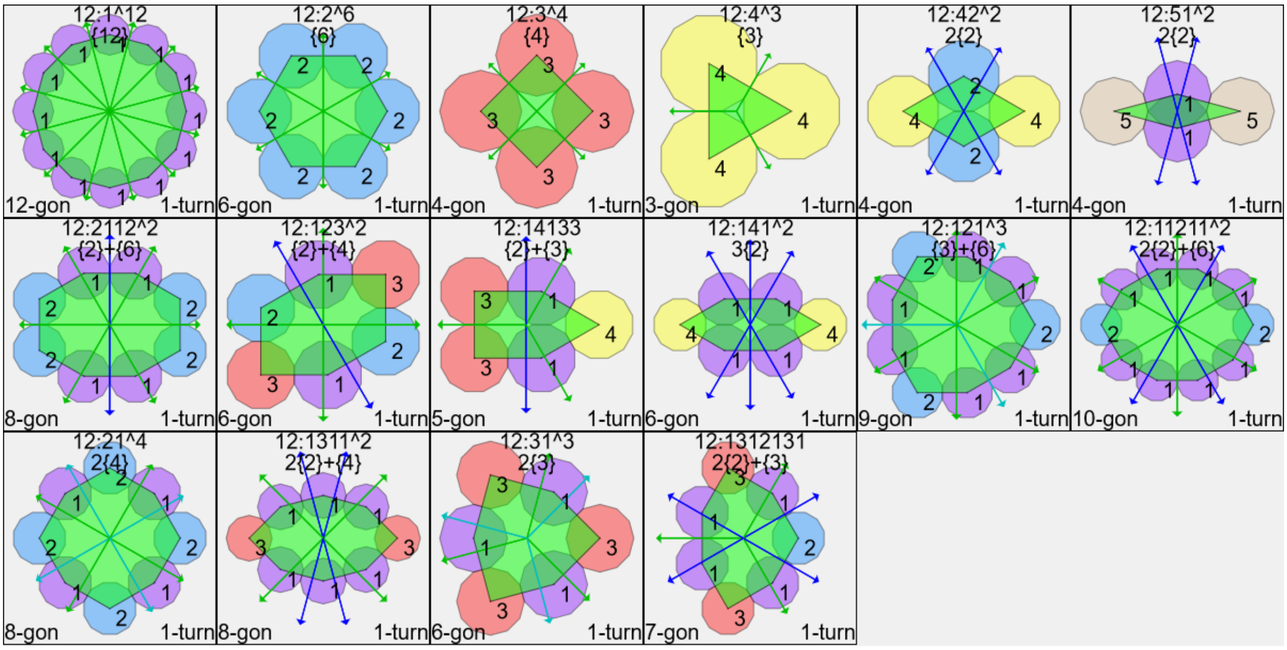


Convex Decatiles

#	Notations		n	m	v	Sym	k-belts			Minkowski Sum	Dim
	Cyclic	Dihedral					2	5	10		
1	10:1^10	10:|1||^10	10	1	10	r20	0	0	1	{10}	2
2	10:2^5	10:|2||^5	5	1	5	r10	0	1	0	{5}	2
3	10:32^2	10:|3||2|^2	2	2	4	d4	2	0	0	2{2}	2
4	10:41^2	10:|4||1|^2	2	2	4	d4	2	0	0	2{2}	2
5	10:221^2	10:|1|2||^2	2	3	6	i4	3	0	0	3{2}	3
6	10:311^2	10:|3|1||^2	2	3	6	i4	3	0	0	3{2}	3
7	10:2111^2	10:|2|1|1|^2	2	4	8	d4	4	0	0	4{2}	4

Convex Dodecatiles

There are 16 convex dodecatiles, including a regular dodecagon, hexagon, square, triangle, and 2 rhombi. The 12 irregulars are decomposed into Minkowski sums of {2}, {3}, {4], {6}. All but one has dihedral symmetry.

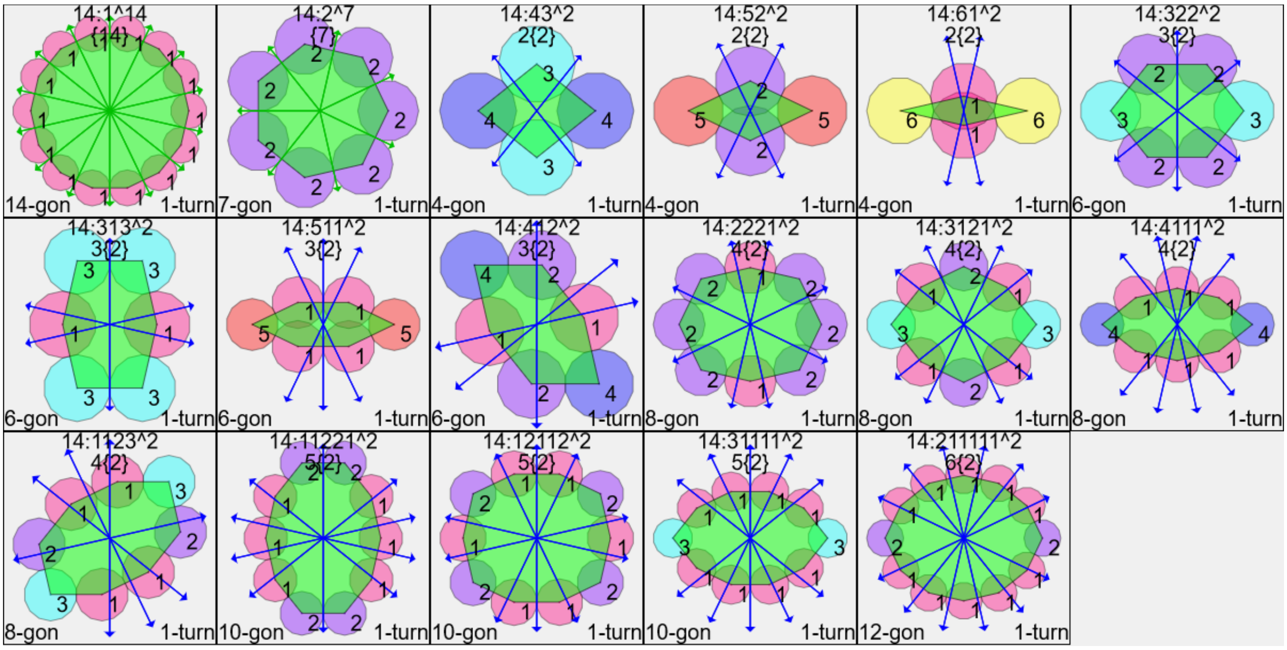


Convex Dodecatiles

#	Notations		n	m	V	Sym	k-belts					Minkowski Sum	Dim
	Cyclic	Dihedral					2	3	4	6	12		
1	12:1^12	12:|1||^12	12	1	12	r24	0	0	0	0	1	{12}	2
2	12:2^6	12:|2||^6	6	1	6	r12	0	0	0	1	0	{6}	2
3	12:3^4	12:|3||^4	4	1	4	r8	0	0	1	0	0	{4}	2
4	12:4^3	12:|4||^3	3	1	3	r6	0	1	0	0	0	{3}	2
5	12:42^2	12:|4||2|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
6	12:51^2	12:|5||1|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
7	12:2112^2	12:||2.1||^2	2	4	8	p4	1	0	0	1	0	{2}+{6}	3
8	12:123^2		2	3	6	g2	1	0	1	0	0	{2}+{4}	3
9	12:14133	12:|4|1.3||	1	5	5	i2	1	1	0	0	0	{2}+{3}	3
10	12:141^2	12:|4|1||^2	2	3	6	i4	3	0	0	0	0	3{2}	3
11	12:121^3	12:|2|1||^3	3	3	9	i6	0	1	0	1	0	{3}+{6}	4
12	12:11211^2	12:|2|1^2||^2	2	5	10	i4	2	0	0	1	0	2{2}+{6}	4
13	12:21^4	12:|2||1|^4	4	2	8	d8	0	0	2	0	0	2{4}	4
14	12:1311^2	12:|3|1|1|^2	2	4	8	d4	2	0	1	0	0	2{2}+{4}	4
15	12:31^3	12:|3||1|^3	3	2	6	d6	0	2	0	0	0	2{3}	4
16	12:1312131	12:|2|1.3.1||	1	7	7	i2	2	1	0	0	0	2{2}+{3}	4

Convex Tetradecatiles

There are 17 convex tetradecatiles, including the regular 14-gon, 7-gon, and 3 rhombi. All except the 7-gon are zonogons. All but two have dihedral symmetry.

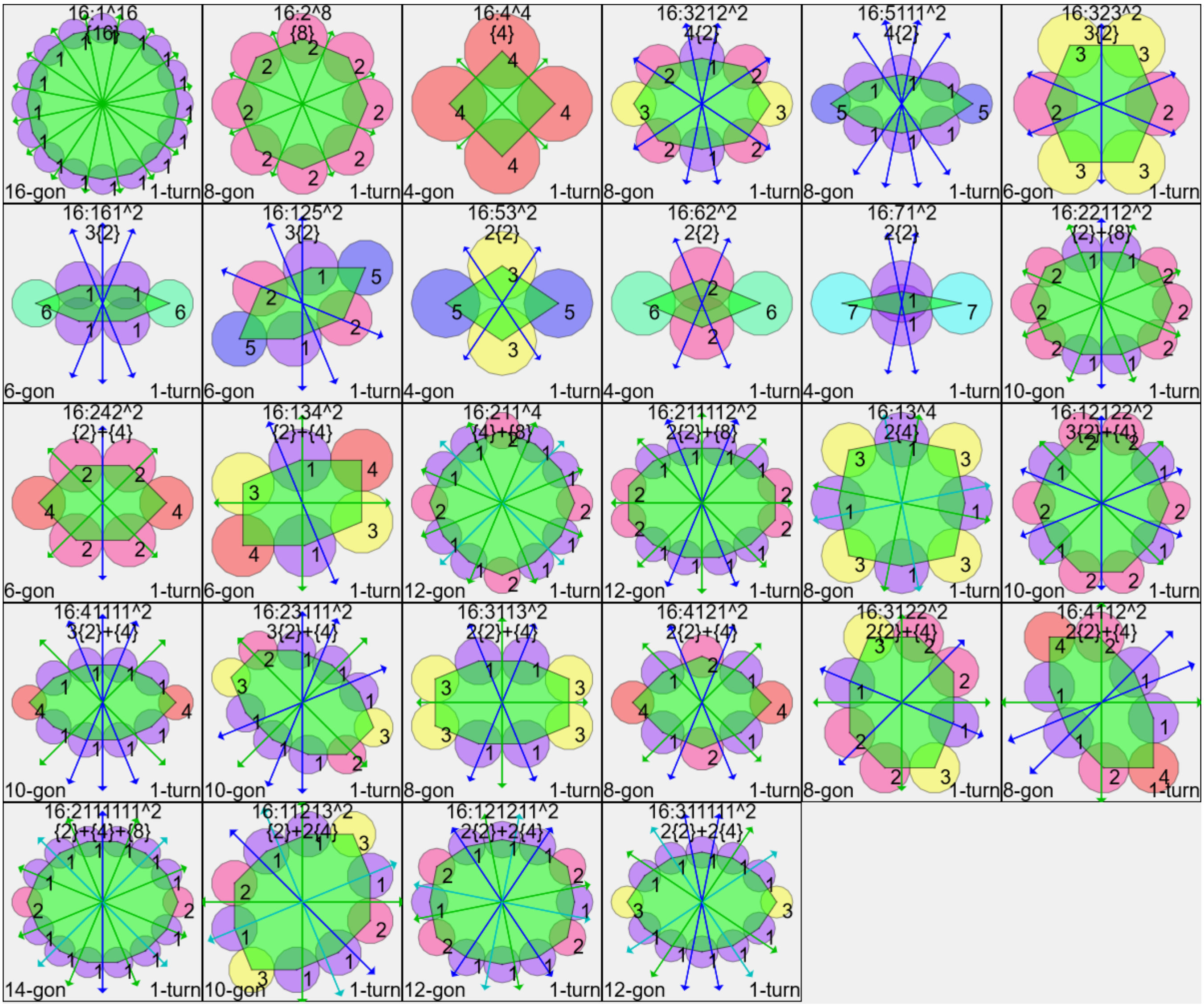


Convex 14-tiles

#	Notations		n	m	v	Sym	k-belts			Minkowski	Dim
	Cyclic	Dihedral					2	7	14	Sum	
1	14:1^14	14:|1||^14	14	1	14	r28	0	0	1	{14}	2
2	14:2^7	14:|2||^7	7	1	7	r14	0	1	0	{7}	2
3	14:43^2	14:|4||3|^2	2	2	4	d4	2	0	0	2{2}	2
4	14:52^2	14:|5||2|^2	2	2	4	d4	2	0	0	2{2}	2
5	14:61^2	14:|6||1|^2	2	2	4	d4	2	0	0	2{2}	2
6	14:322^2	14:|3|2||^2	2	3	6	i4	3	0	0	3{2}	3
7	14:313^2	14:|1|3||^2	2	3	6	i4	3	0	0	3{2}	3
8	14:511^2	14:|5|1||^2	2	3	6	i4	3	0	0	3{2}	3
9	14:412^2		2	3	6	g2	3	0	0	3{2}	3
10	14:2221^2	14:|2|2|1|^2	2	4	8	d4	4	0	0	4{2}	4
11	14:3121^2	14:|3|1|2|^2	2	4	8	d4	4	0	0	4{2}	4
12	14:4111^2	14:|4|1|1|^2	2	4	8	d4	4	0	0	4{2}	4
13	14:1123^2		2	4	8	g2	4	0	0	4{2}	4
14	14:11221^2	14:|1|1.2||^2	2	5	10	i4	5	0	0	5{2}	5
15	14:12112^2	14:|1|2.1||^2	2	5	10	i4	5	0	0	5{2}	5
16	14:31111^2	14:|3|1^2||^2	2	5	10	i4	5	0	0	5{2}	5
17	14:211111^2	14:|2|1^2|1|^2	2	6	12	d4	6	0	0	6{2}	6

Convex Hexadecatiles

There are 28 convex hexadecatiles, including a regular 16-gon, octagon, square, along with 3 other rhombi.

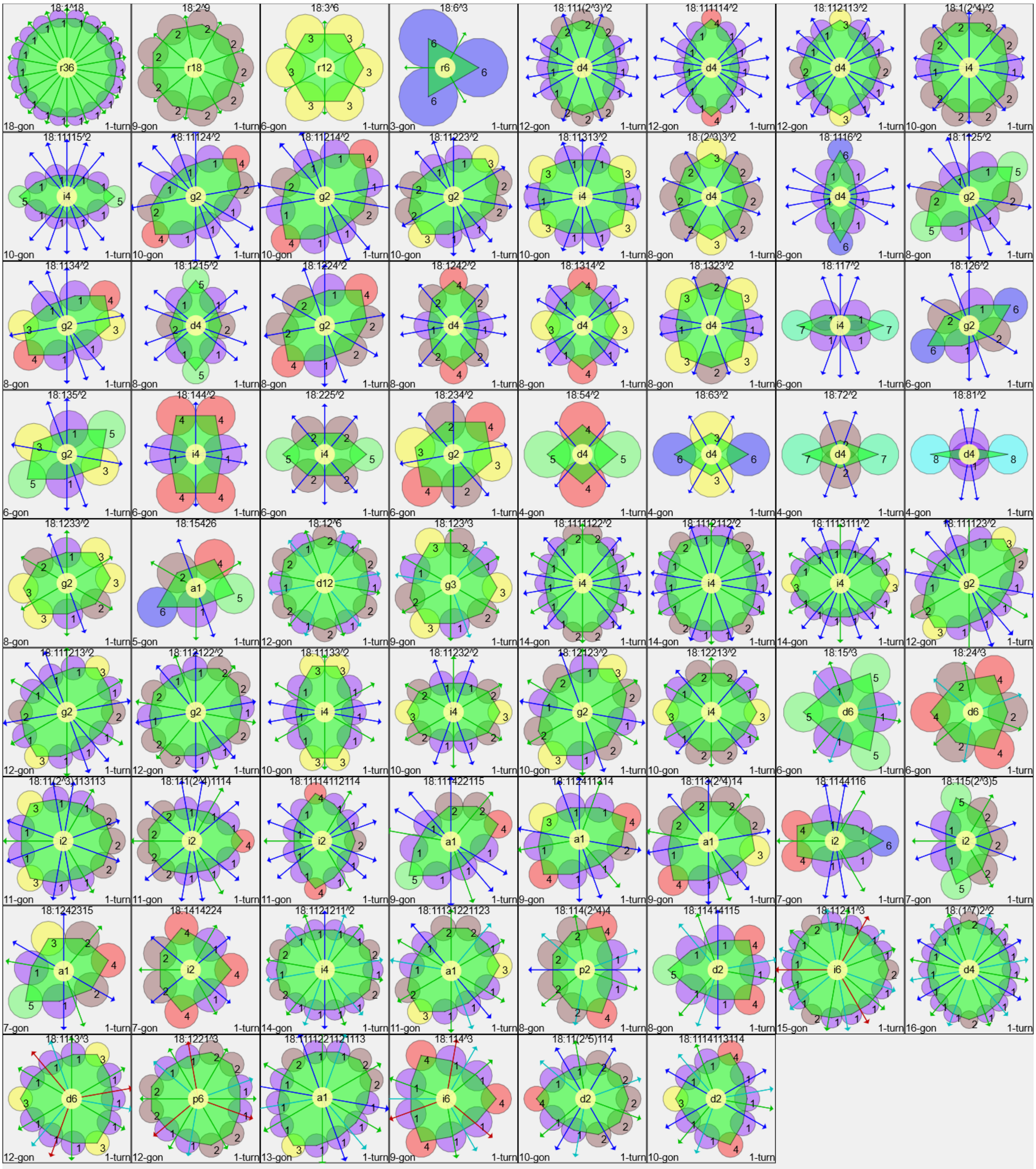


Convex 16-tiles

#	Notations		n	m	v	Sym	k-belts				Minkowski Sum	Dim
	Cyclic	Dihedral					2	4	8	16		
1	16:1^16	16:|1||^16	16	1	16	r32	0	0	0	1	{16}	2
2	16:2^8	16:|2||^8	8	1	8	r16	0	0	1	0	{8}	2
3	16:4^4	16:|4||^4	4	1	4	r8	0	1	0	0	{4}	2
4	16:3212^2	16:|3|2|1|^2	2	4	8	d4	4	0	0	0	4{2}	4
5	16:5111^2	16:|5|1|1|^2	2	4	8	d4	4	0	0	0	4{2}	4
6	16:323^2	16:|2|3||^2	2	3	6	i4	3	0	0	0	3{2}	3
7	16:161^2	16:|6|1||^2	2	3	6	i4	3	0	0	0	3{2}	3
8	16:125^2		2	3	6	g2	3	0	0	0	3{2}	3
9	16:53^2	16:|5||3|^2	2	2	4	d4	2	0	0	0	2{2}	2
10	16:62^2	16:|6||2|^2	2	2	4	d4	2	0	0	0	2{2}	2
11	16:71^2	16:|7||1|^2	2	2	4	d4	2	0	0	0	2{2}	2
12	16:22112^2	16:|2|2.1||^2	2	5	10	i4	1	0	1	0	{2}+{8}	3
13	16:242^2	16:|4|2||^2	2	3	6	i4	1	1	0	0	{2}+{4}	3
14	16:134^2		2	3	6	g2	1	1	0	0	{2}+{4}	3
15	16:211^4	16:|2|1||^4	4	3	12	i8	0	1	1	0	{4}+{8}	4
16	16:211112^2	16:||2.1.1||^2	2	6	12	p4	2	0	1	0	2{2}+{8}	4
17	16:13^4	16:|1||3|^4	4	2	8	d8	0	2	0	0	2{4}	4
18	16:12122^2	16:|2|1.2||^2	2	5	10	i4	3	1	0	0	3{2}+{4}	5
19	16:41111^2	16:|4|1^2||^2	2	5	10	i4	3	1	0	0	3{2}+{4}	5
20	16:23111^2		2	5	10	g2	3	1	0	0	3{2}+{4}	5
21	16:3113^2	16:||3.1||^2	2	4	8	p4	2	1	0	0	2{2}+{4}	4
22	16:4121^2	16:|4|1|2|^2	2	4	8	d4	2	1	0	0	2{2}+{4}	4
23	16:3122^2		2	4	8	g2	2	1	0	0	2{2}+{4}	4
24	16:4112^2		2	4	8	g2	2	1	0	0	2{2}+{4}	4
25	16:2111111^2	16:|2|1^3||^2	2	7	14	i4	1	1	1	0	{2}+{4}+{8}	5
26	16:11213^2		2	5	10	g2	1	2	0	0	{2}+2{4}	5
27	16:121211^2	16:|1|2.1|1|^2	2	6	12	d4	2	2	0	0	2{2}+2{4}	6
28	16:311111^2	16:|3|1^2|1|^2	2	6	12	d4	2	2	0	0	2{2}+2{4}	6

Convex Octadecatiles

There are 70 convex octadecatiles, including the regular 18-gon, 9-gon, hexagon, triangle, and 4 rhombi.



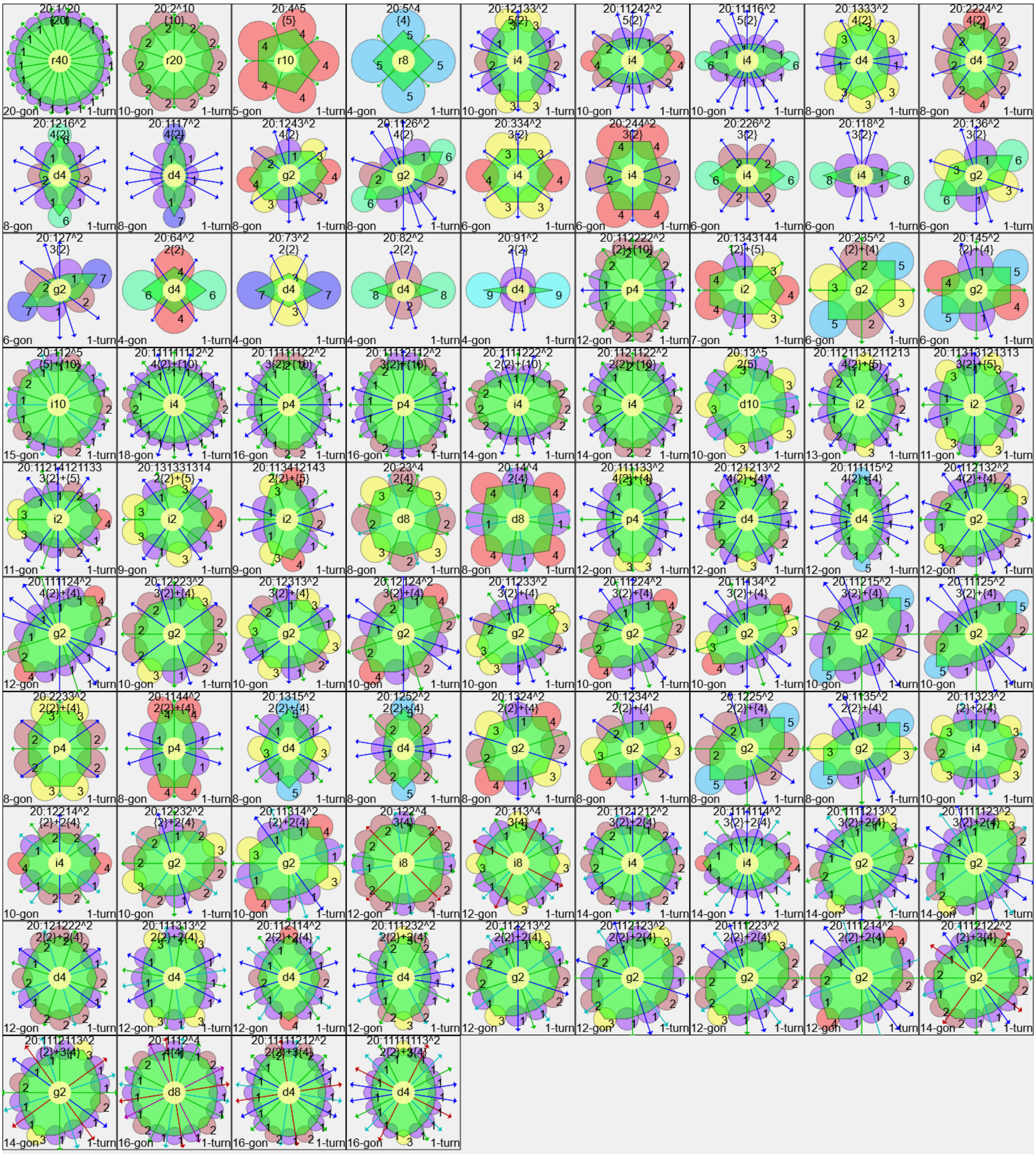
Convex 18-tiles

#	Notations		n	m	v	Sym	k-belts					Minkowski Sum	Dim
	Cyclic	Dihedral					2	3	6	9	18		
1	18:1^18	18:|1|||^18	18	1	18	r36	0	0	0	0	1	{18}	2
2	18:2^9	18:|2|||^9	9	1	9	r18	0	0	0	1	0	{9}	2
3	18:3^6	18:|3|||^6	6	1	6	r12	0	0	1	0	0	{6}	2
4	18:6^3	18:|6|||^3	3	1	3	r6	0	1	0	0	0	{3}	2
5	18:111(2^3)^2	18:|1|1.2|2|^2	2	6	12	d4	6	0	0	0	0	6{2}	6
6	18:111114^2	18:|1|1^2|4|^2	2	6	12	d4	6	0	0	0	0	6{2}	6
7	18:112113^2	18:|2|1^2|3|^2	2	6	12	d4	6	0	0	0	0	6{2}	6
8	18:1(2^4)^2	18:|1|2^2||^2	2	5	10	i4	5	0	0	0	0	5{2}	5
9	18:11115^2	18:|5|1^2||^2	2	5	10	i4	5	0	0	0	0	5{2}	5
10	18:11124^2		2	5	10	g2	5	0	0	0	0	5{2}	5
11	18:11214^2		2	5	10	g2	5	0	0	0	0	5{2}	5
12	18:11223^2		2	5	10	g2	5	0	0	0	0	5{2}	5
13	18:11313^2	18:|1|3.1||^2	2	5	10	i4	5	0	0	0	0	5{2}	5
14	18:(2^3)3^2	18:|2|2|3|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
15	18:1116^2	18:|1|1|6|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
16	18:1125^2		2	4	8	g2	4	0	0	0	0	4{2}	4
17	18:1134^2		2	4	8	g2	4	0	0	0	0	4{2}	4
18	18:1215^2	18:|2|1|5|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
19	18:1224^2		2	4	8	g2	4	0	0	0	0	4{2}	4
20	18:1242^2	18:|1|2|4|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
21	18:1314^2	18:|3|1|4|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
22	18:1323^2	18:|1|3|2|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
23	18:117^2	18:|7|1||^2	2	3	6	i4	3	0	0	0	0	3{2}	3
24	18:126^2		2	3	6	g2	3	0	0	0	0	3{2}	3
25	18:135^2		2	3	6	g2	3	0	0	0	0	3{2}	3
26	18:144^2	18:|1|4||^2	2	3	6	i4	3	0	0	0	0	3{2}	3
27	18:225^2	18:|5|2||^2	2	3	6	i4	3	0	0	0	0	3{2}	3
28	18:234^2		2	3	6	g2	3	0	0	0	0	3{2}	3
29	18:54^2	18:|5||4|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
30	18:63^2	18:|6||3|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
31	18:72^2	18:|7||2|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
32	18:81^2	18:|8||1|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
33	18:1233^2		2	4	8	g2	1	0	1	0	0	{2}+{6}	3
34	18:15426		1	5	5	a1	1	1	0	0	0	{2}+{3}	3
35	18:12^6	18:|1||2|^6	6	2	12	d12	0	0	2	0	0	2{6}	4
36	18:123^3		3	3	9	g3	0	1	1	0	0	{3}+{6}	4
37	18:1111122^2	18:|1|1.1.2||^2	2	7	14	i4	4	0	1	0	0	4{2}+{6}	6
38	18:1112112^2	18:|1|1.2.1||^2	2	7	14	i4	4	0	1	0	0	4{2}+{6}	6
39	18:1113111^2	18:|3|1^3||^2	2	7	14	i4	4	0	1	0	0	4{2}+{6}	6
40	18:111123^2		2	6	12	g2	3	0	1	0	0	3{2}+{6}	5
41	18:111213^2		2	6	12	g2	3	0	1	0	0	3{2}+{6}	5
42	18:112122^2		2	6	12	g2	3	0	1	0	0	3{2}+{6}	5

43	18:11133^2	18:|1|1.3||^2	2	5	10	i4	2	0	1	0	0	2{2}+{6}	4
44	18:11232^2	18:|3|2.1||^2	2	5	10	i4	2	0	1	0	0	2{2}+{6}	4
45	18:12123^2		2	5	10	g2	2	0	1	0	0	2{2}+{6}	4
46	18:12213^2	18:|3|1.2||^2	2	5	10	i4	2	0	1	0	0	2{2}+{6}	4
47	18:15^3	18:|1||5|^3	3	2	6	d6	0	2	0	0	0	2{3}	4
48	18:24^3	18:|2||4|^3	3	2	6	d6	0	2	0	0	0	2{3}	4
49	18:11(2^3)113113	18:|2|2.1.1.3.1||	1	11	11	i2	4	1	0	0	0	4{2}+{3}	6
50	18:111(2^4)1114	18:|4|1.1.1.2.2||	1	11	11	i2	4	1	0	0	0	4{2}+{3}	6
51	18:11114112114	18:|2|1.1.4.1.1||	1	11	11	i2	4	1	0	0	0	4{2}+{3}	6
52	18:111422115		1	9	9	a1	3	1	0	0	0	3{2}+{3}	5
53	18:112411314		1	9	9	a1	3	1	0	0	0	3{2}+{3}	5
54	18:113(2^4)14		1	9	9	a1	3	1	0	0	0	3{2}+{3}	5
55	18:1144116	18:|6|1.1.4||	1	7	7	i2	2	1	0	0	0	2{2}+{3}	4
56	18:115(2^3)5	18:|2|2.5.1||	1	7	7	i2	2	1	0	0	0	2{2}+{3}	4
57	18:1242315		1	7	7	a1	2	1	0	0	0	2{2}+{3}	4
58	18:1414224	18:|4|1.4.2||	1	7	7	i2	2	1	0	0	0	2{2}+{3}	4
59	18:1121211^2	18:|1|2.1.1||^2	2	7	14	i4	1	0	2	0	0	{2}+2{6}	5
60	18:11131221123		1	11	11	a1	1	1	1	0	0	{2}+{3}+{6}	5
61	18:114(2^4)4	18:||1.4.2.2||	1	8	8	p2	1	2	0	0	0	{2}+2{3}	5
62	18:11414115	18:|1|4.1.1|5|	1	8	8	d2	1	2	0	0	0	{2}+2{3}	5
63	18:11211^3	18:|2|1^2||^3	3	5	15	i6	0	1	2	0	0	{3}+2{6}	6
64	18:(1^7)2^2	18:|1|1^3|2|^2	2	8	16	d4	2	0	2	0	0	2{2}+2{6}	6
65	18:1113^3	18:|1|1|3|^3	3	4	12	d6	0	2	1	0	0	2{3}+{6}	6
66	18:1221^3	18:||1.2||^3	3	4	12	p6	0	2	1	0	0	2{3}+{6}	6
67	18:1111221121113		1	13	13	a1	2	1	1	0	0	2{2}+{3}+{6}	6
68	18:114^3	18:|4|1||^3	3	3	9	i6	0	3	0	0	0	3{3}	6
69	18:11(2^5)114	18:|2|2.2.1.1|4|	1	10	10	d2	2	2	0	0	0	2{2}+2{3}	6
70	18:1114113114	18:|1|1.4.1.1|3|	1	10	10	d2	2	2	0	0	0	2{2}+2{3}	6

Convex icosatiles

There are 85 convex icosatiles, including the regular decagon, pentagon, and square.



Convex 20-tiles

#	Notations		n	m	v	Sym	k-belts					Minkowski	Dim
	Cyclic	Dihedral					2	4	5	10	20	Sum	
1	20:1^20	20:|1||^20	20	1	20	r40	0	0	0	0	1	{20}	2
2	20:2^10	20:|2||^10	10	1	10	r20	0	0	0	1	0	{10}	2
3	20:4^5	20:|4||^5	5	1	5	r10	0	0	1	0	0	{5}	2
4	20:5^4	20:|5||^4	4	1	4	r8	0	1	0	0	0	{4}	2
5	20:12133^2	20:|2|1.3||^2	2	5	10	i4	5	0	0	0	0	5{2}	5
6	20:11242^2	20:|4|2.1||^2	2	5	10	i4	5	0	0	0	0	5{2}	5
7	20:11116^2	20:|6|1^2||^2	2	5	10	i4	5	0	0	0	0	5{2}	5
8	20:1333^2	20:|1|3|3|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
9	20:2224^2	20:|2|2|4|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
10	20:1216^2	20:|2|1|6|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
11	20:1117^2	20:|1|1|7|^2	2	4	8	d4	4	0	0	0	0	4{2}	4
12	20:1243^2		2	4	8	g2	4	0	0	0	0	4{2}	4
13	20:1126^2		2	4	8	g2	4	0	0	0	0	4{2}	4
14	20:334^2	20:|4|3||^2	2	3	6	i4	3	0	0	0	0	3{2}	3
15	20:244^2	20:|2|4||^2	2	3	6	i4	3	0	0	0	0	3{2}	3
16	20:226^2	20:|6|2||^2	2	3	6	i4	3	0	0	0	0	3{2}	3
17	20:118^2	20:|8|1||^2	2	3	6	i4	3	0	0	0	0	3{2}	3
18	20:136^2		2	3	6	g2	3	0	0	0	0	3{2}	3
19	20:127^2		2	3	6	g2	3	0	0	0	0	3{2}	3
20	20:64^2	20:|6||4|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
21	20:73^2	20:|7||3|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
22	20:82^2	20:|8||2|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
23	20:91^2	20:|9||1|^2	2	2	4	d4	2	0	0	0	0	2{2}	2
24	20:112222^2	20:||1.2.2||^2	2	6	12	p4	1	0	0	1	0	{2}+{10}	3
25	20:1343144	20:|4|3.1.4||	1	7	7	i2	1	0	1	0	0	{2}+{5}	3
26	20:235^2		2	3	6	g2	1	1	0	0	0	{2}+{4}	3
27	20:145^2		2	3	6	g2	1	1	0	0	0	{2}+{4}	3
28	20:112^5	20:|2|1||^5	5	3	15	i10	0	0	1	1	0	{5}+{10}	4
29	20:111111112^2	20:|2|1^4||^2	2	9	18	i4	4	0	0	1	0	4{2}+{10}	6
30	20:11111122^2	20:||1.1.1.2||^2	2	8	16	p4	3	0	0	1	0	3{2}+{10}	5
31	20:11112112^2	20:||1.1.2.1||^2	2	8	16	p4	3	0	0	1	0	3{2}+{10}	5
32	20:1111222^2	20:|2|2.1.1||^2	2	7	14	i4	2	0	0	1	0	2{2}+{10}	4
33	20:1121122^2	20:|2|1.1.2||^2	2	7	14	i4	2	0	0	1	0	2{2}+{10}	4
34	20:13^5	20:|1||3|^5	5	2	10	d10	0	0	2	0	0	2{5}	4
35	20:1121131211213	20:|2|1.1.3.1.2.1||	1	13	13	i2	4	0	1	0	0	4{2}+{5}	6
36	20:11313121313	20:|2|1.3.1.3.1||	1	11	11	i2	3	0	1	0	0	3{2}+{5}	5
37	20:11214121133	20:|4|1.2.1.1.3||	1	11	11	i2	3	0	1	0	0	3{2}+{5}	5
38	20:131331314	20:|4|1.3.1.3||	1	9	9	i2	2	0	1	0	0	2{2}+{5}	4
39	20:113412143	20:|2|1.4.3.1||	1	9	9	i2	2	0	1	0	0	2{2}+{5}	4
40	20:23^4	20:|2||3|^4	4	2	8	d8	0	2	0	0	0	2{4}	4
41	20:14^4	20:|1||4|^4	4	2	8	d8	0	2	0	0	0	2{4}	4
42	20:111133^2	20:||1.1.3||^2	2	6	12	p4	4	1	0	0	0	4{2}+{4}	6

43	20:121213^2	20:|1|2.1|3|^2	2	6	12	d4	4	1	0	0	0	4{2}+{4}	6
44	20:111115^2	20:|1|1^2|5|^2	2	6	12	d4	4	1	0	0	0	4{2}+{4}	6
45	20:112132^2		2	6	12	g2	4	1	0	0	0	4{2}+{4}	6
46	20:111124^2		2	6	12	g2	4	1	0	0	0	4{2}+{4}	6
47	20:12223^2		2	5	10	g2	3	1	0	0	0	3{2}+{4}	5
48	20:12313^2		2	5	10	g2	3	1	0	0	0	3{2}+{4}	5
49	20:12124^2		2	5	10	g2	3	1	0	0	0	3{2}+{4}	5
50	20:11233^2		2	5	10	g2	3	1	0	0	0	3{2}+{4}	5
51	20:11224^2		2	5	10	g2	3	1	0	0	0	3{2}+{4}	5
52	20:11134^2		2	5	10	g2	3	1	0	0	0	3{2}+{4}	5
53	20:11215^2		2	5	10	g2	3	1	0	0	0	3{2}+{4}	5
54	20:11125^2		2	5	10	g2	3	1	0	0	0	3{2}+{4}	5
55	20:2233^2	20:||2.3||^2	2	4	8	p4	2	1	0	0	0	2{2}+{4}	4
56	20:1144^2	20:||1.4||^2	2	4	8	p4	2	1	0	0	0	2{2}+{4}	4
57	20:1315^2	20:|3|1|5|^2	2	4	8	d4	2	1	0	0	0	2{2}+{4}	4
58	20:1252^2	20:|1|2|5|^2	2	4	8	d4	2	1	0	0	0	2{2}+{4}	4
59	20:1324^2		2	4	8	g2	2	1	0	0	0	2{2}+{4}	4
60	20:1234^2		2	4	8	g2	2	1	0	0	0	2{2}+{4}	4
61	20:1225^2		2	4	8	g2	2	1	0	0	0	2{2}+{4}	4
62	20:1135^2		2	4	8	g2	2	1	0	0	0	2{2}+{4}	4
63	20:11323^2	20:|2|3.1||^2	2	5	10	i4	1	2	0	0	0	{2}+2{4}	5
64	20:12214^2	20:|4|1.2||^2	2	5	10	i4	1	2	0	0	0	{2}+2{4}	5
65	20:12232^2		2	5	10	g2	1	2	0	0	0	{2}+2{4}	5
66	20:11314^2		2	5	10	g2	1	2	0	0	0	{2}+2{4}	5
67	20:122^4	20:|1|2||^4	4	3	12	i8	0	3	0	0	0	3{4}	6
68	20:113^4	20:|3|1||^4	4	3	12	i8	0	3	0	0	0	3{4}	6
69	20:1121212^2	20:|2|1.2.1||^2	2	7	14	i4	3	2	0	0	0	3{2}+2{4}	7
70	20:1111114^2	20:|4|1^3||^2	2	7	14	i4	3	2	0	0	0	3{2}+2{4}	7
71	20:1111213^2		2	7	14	g2	3	2	0	0	0	3{2}+2{4}	7
72	20:1111123^2		2	7	14	g2	3	2	0	0	0	3{2}+2{4}	7
73	20:121222^2	20:|2|1.2|2|^2	2	6	12	d4	2	2	0	0	0	2{2}+2{4}	6
74	20:111313^2	20:|1|1.3|1|^2	2	6	12	d4	2	2	0	0	0	2{2}+2{4}	6
75	20:112114^2	20:|2|1^2|4|^2	2	6	12	d4	2	2	0	0	0	2{2}+2{4}	6
76	20:111232^2	20:|1|1.2|3|^2	2	6	12	d4	2	2	0	0	0	2{2}+2{4}	6
77	20:112213^2		2	6	12	g2	2	2	0	0	0	2{2}+2{4}	6
78	20:112123^2		2	6	12	g2	2	2	0	0	0	2{2}+2{4}	6
79	20:111223^2		2	6	12	g2	2	2	0	0	0	2{2}+2{4}	6
80	20:111214^2		2	6	12	g2	2	2	0	0	0	2{2}+2{4}	6
81	20:1112122^2		2	7	14	g2	1	3	0	0	0	{2}+3{4}	7
82	20:1112113^2		2	7	14	g2	1	3	0	0	0	{2}+3{4}	7
83	20:1112^4	20:|1|1|2|^4	4	4	16	d8	0	4	0	0	0	4{4}	8
84	20:11111212^2	20:|1|1.1.2|1|^2	2	8	16	d4	2	3	0	0	0	2{2}+3{4}	8
85	20:11111113^2	20:|1|1^3|3|^2	2	8	16	d4	2	3	0	0	0	2{2}+3{4}	8

6 Higher convex polytile tables by symmetry

These tables of convex p-tiles and higher are available online from the "Convex polytile" sets in the Polytile Explorer via the "Set Selection" and "Report" button. Clicking on links will show that subset of tiles.

Odd-sided polygons are given a green background.

The number of p-tiles as p increases becomes unwieldy as a complete set, but subsets of tiles may be of interest for tiling projects, and knowing them by symmetry and sides can help identify tiles of interest.

Tile counts by sides and symmetry

Tetratiles, Hexatiles, Octatiles

4

Sides	r8	TT
4	1	1
Total	1	1

6

Sides	r12	r6	d4	TT
3		1		1
4			1	1
6	1			1
Total	1	1	1	3

8

Sides	r16	r8	i4	d4	TT
4		1		1	2
6			1		1
8	1				1
Total	1	1	1	1	4

Decatiles and Dodecatiles

10

Sides	r20	r10	i4	d4	TT
4				2	2
5		1			1
6			2		2
8				1	1
10	1				1
Total	1	1	2	3	7

12

Sides	r24	r12	r8	r6	p4	i6	i4	i2	d8	d6	d4	g2	TT
3				1									1
4			1								2		3
5								1					1
6		1					1			1		1	4
7								1					1
8					1				1		1		3
9						1							1
10							1						1
12	1												1
Total	1	1	1	1	1	1	2	2	1	1	3	1	16

Tetradecatiles and Hexadecatiles

14

Sides	r28	r14	i4	d4	g2	TT
4				3		3
6			3		1	4
7		1				1
8				3	1	4
10			3			3
12				1		1
14	1					1
Total	1	1	6	7	2	17

16

Sides	r32	r16	r8	p4	i8	i4	d8	d4	g2	TT
4			1					3		4
6						3			2	5
8		1		1			1	3	2	8
10						3			2	5
12				1	1			2		4
14						1				1
16	1									1
Total	1	1	1	2	1	7	1	8	6	28

Octadecatiles

Sides	r36	r18	r12	r6	p6	p2	i6	i4	i2	d12	d6	d4	d2	g3	g2	a1	TT
3				1													1
4												4					4
5																1	1
6			1					3			2				3		9
7									3							1	4
8						1						6	1		4		12
9		1					1							1		3	6
10								6					2		4		12
11									3							1	4
12					1					1	1	3			3		9
13																1	1
14								4									4
15							1										1
16												1					1
18	1																1
Total	1	1	1	1	1	1	2	13	6	1	3	14	3	1	14	7	70

Icosatiles and 22-tiles

20

Sides	r40	r20	r10	r8	p4	i10	i8	i4	i2	d10	d8	d4	g2	TT
4				1								4		5
5			1											1
6								4					4	8
7									1					1
8					2						2	6	6	16
9									2					2
10		1						5		1			10	17
11									2					2
12					2		2					6	6	16
13									1					1
14								4					4	8
15						1								1
16					2						1	2		5
18								1						1
20	1													1
Total	1	1	1	1	6	1	2	14	6	1	3	18	30	85

22

Sides	r44	r22	i4	d4	g2	TT
4				5		5
6			5		5	10
8				10	10	20
10			10		16	26
11		1				1
12				10	16	26
14			10		10	20
16				5	5	10
18			5			5
20				1		1
22	1					1
Total	1	1	30	31	62	125

24-tiles

Sides	r48	r24	r16	r12	r8	p8	p6	p4	p2	i12	i8	i6	i4	i2	d16	d12	d8	d6	d4	d2	g4	g3	g2	a1	TT
4					1														5						6
5														1										1	2
6				1									4					3					7		15
7														4										4	8
8			1					2	1								2		10	1			14	2	33
9												3		3								2		12	20
10									1				10							8			28	3	50
11														8										19	27
12		1					1	4	1		1					1		3	13	1	1	2	29	8	66
13														8										19	27
14									2				10							7			28	3	50
15												3		3								2		12	20
16						1		4	1						1		1		8	1			14	2	33
17														4										4	8
18							1			1			4					2					7		15
19														1										1	2
20								2			1								3						6
21												1													1
22													1												1
24	1																								1
Total	1	1	1	1	1	1	2	12	6	1	2	7	29	32	1	1	3	8	39	18	1	6	127	90	391

26-tiles and 28-tiles

26

Sides	r52	r26	i4	d4	g2	TT
4				6		6
6			6		8	14
8				15	20	35
10			15		42	57
12				20	56	76
13		1				1
14			20		56	76
16				15	42	57
18			15		20	35
20				6	8	14
22			6			6
24				1		1
26	1					1
Total	1	1	62	63	252	379

28

Sides	r56	r28	r8	p4	i8	i4	d8	d4	g4	g2	Total
4			1					6			7
6						6				10	16
8				3			3	15		26	47
10						15				64	79
12				6	3			26	1	90	126
14		1				19				113	133
16				10			3	22	1	90	126
18						15				64	79
20				6	3			12		26	47
22						6				10	16
24				3			1	3			7
26						1					1
28	1										1
Total	1	1	1	28	6	62	7	84	2	493	685

30-tiles

Side	r60	r30	r20	r12	r10	r6	p10	p6	p2	i12	i10	i6	i4	i2	d20	d12	d10	d6	d4	d2	g5	g3	g2	a1	Total
3						1																			1
4																			7						7
5					1																			2	3
6				1									6					4		1			12		24
7														8										9	17
8									3										21	4			35	8	71
9												4		6								4		46	60
10			1						2				20				2			20			90	38	**173**
11														23										122	**145**
12								2	11							2		6	33	13		6	150	106	**329**
13														25										224	**249**
14									8				35							36			197	166	**442**
15		1									1	5		30							1	10		267	**315**
16									20										35	24			197	166	**442**
17														25										224	**249**
18								2	6	2			33					6		18		6	150	106	**329**
19														23										122	**145**
20							1		12						1		1		20	10			90	38	**173**
21												4		6								4		46	**60**
22									2				21							5			35	8	**71**
23														8										9	17
24								2	1							1		2	6				12		24
25											1													2	3
26													7												7
27												1													1
28																			1						1
30	1																								1
TT	1	1	1	1	1	1	1	6	**65**	2	2	14	**122**	**154**	1	3	3	18	**123**	**131**	1	30	**968**	**1709**	**3359**

5.9 32-tiles

Sides	r64	r32	r16	r8	p8	p4	i16	i8	i4	d16	d8	d4	g4	g2	Total
4				1								7			8
6									7					14	21
8			1			3					3	21		44	72
10									21					126	**147**
12						9		3				44	2	222	**280**
14									35					340	**375**
16		1			1	16				1	3	48	2	368	**440**
18									35					340	**375**
20						16		3				37	2	222	**280**
22									21					126	**147**
24					1	9	1				2	15		44	**72**
26									7					14	21
28						3		1				4			8
30									1						1
32	1														1
Total	1	1	1	1	2	56	1	7	**127**	1	8	**176**	6	**1860**	**2248**

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